Total No. of Printed Pages:2

SUBJECT CODE NO:- B-2037
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-VI) Examination March/April 2018
Mathematics MAT -603
1 Mathematical Statistics-II - 603

[Time: 1:30 Hours] **[Max.Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) If X is random variable, then prove that $V(ax+b)=a^2v(x)$ where a and b are constants. Also prove that variance is independent of change of origin and scale

b) If $\mu_r{}'$exists then prove that $\mu_s{}'$ exists for all $1 \le s \le r$

B) Attempt any one: 07

c) A coin is tossed until a head appears. What is the expectation of the number of tosses required?

d) A box contains 'a' white and 'b' black balls. 'C' balls are drawn. Find the expected value of the number of white balls drawn.

Q.2 A) Attempt any one: 08

a) Obtain the moment generating function of the binomial distribution and hence find variance, μ_3 and μ_4.

b) Obtain the moment generating function of geometric distribution and hence find mean and variance of geometric distribution.

B) Attempt any one: 07

c) If m things are distributed among 'a' men and 'b' women, show the probability that the number of things received by men is odd and is

$$\frac{1}{2}\left[\frac{(b+a)^m-(b-a)^m}{(b+a)^m}\right]$$

d) Six coins are tossed 6400 times. Using the Poisson distribution, find the approximate probability of getting six heads r times.

Q.3 A) Attempt <u>any one</u>: 05

a) Obtain the cumulant generating function of normal distribution and hence find β_1 and β_2.

b) Prove that correlation coefficient is the geometric mean between the regression coefficient.

B) Attempt <u>any one</u>: 05

c) If X is uniformly distributed with mean 1 and variance $4/3$. Find$P(X < 0)$.

d) Subway trains on a certain line run every half hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes?

Q.4 Choose the correct alternative: 10

1) If X and Y are independent then $COV(X, Y) = ----$
   a) 1
   b) 0
   c) -1
   d) 2

2) For a Poisson distribution mean is 4 the variance is --------
   a) ¼
   b) 2/4
   c) 4
   d) 2

3) The mean and median of normal distribution are --------
   a) The same
   b) Not same
   c) Mean < median
   d) Mean > median

4) For the gamma distribution mean = variance = ------------
   a) 2λ
   b) 6λ
   c) λ
   d) 4/λ

5) The mean of exponential distribution = $\mu_1{}' =$-----------
   a) θ
   b) $1/\theta^2$
   c) θ^2
   d) $1/\theta$

<u>OR</u>

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2037
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-VI) Examination March/April 2018
Mathematics MAT - 604
2 Ordinary Differential Equation-II - 604

[Time: 1:30 Hours] **[Max.Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Figures to the right indicate full marks.

Q.1 A) Attempt <u>any one</u>: 08

a) If $\emptyset_1, ---, \emptyset_n$ are n solutions of $L(y) = y^{(n)} + a_1(x)y^{(n-1)} + ---a_n(x)y = 0$ on an interval I. prove that they are linearly independent there if and only if. $W(\emptyset_1, ---, \emptyset_n)(x) \neq 0$ for all x in I.

b) Let $\emptyset_1, \emptyset_2$ be two solutions of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I, and let x_0 be any point in I. then prove that

$$W(\emptyset_1, \emptyset_2)(x) = \exp\left[-\int_{x_0}^{x} a_1(t)dt\right] W(\emptyset_1, \emptyset_2)(x_0)$$

B) Attempt <u>any one</u>: 07

c) One solution of

$y'' - \frac{2}{x^2}\, y = 0$ on $0 < x < \infty$

Is $\emptyset_1(x) = x^2$. Find all solutions of

$y'' - \frac{2}{x^2}\, y = x$ on $0 < x < \infty$

d) Find two linearly independent power series solution of the equation $y'' + y = 0$

Q.2 A) Attempt <u>any one</u>: 08

a) Show that

$$\int_{-1}^{1} P_n^2(x)dx = \frac{1}{2n+1}$$

b) If $\emptyset_1$ is a solution of
$L(y) = y'' + a_1(x)y' + a_2(x)y = 0$
On an interval I, and $\emptyset_1(x) \neq 0$ on I, prove that the second solution $\emptyset_2$ $of\ L(y) = 0$ on I is given by
$$\emptyset_2(x) = \emptyset_1(x) \int_{x_0}^{x} \frac{1}{[\emptyset_1(s)]^2} \exp\left[-\int_{x_0}^{s} a_1(t)dt\right] ds$$

B) Attempt <u>any one</u>: 07

c) Verify that the function $\emptyset_1(x) = x^3, (x > 0)$ satisfies the equation
$x^2y'' - 7xy' + 15y = 0, 0 < x < \infty$

d) Show that the coefficient of x^n in $P_n(x)$
Is $\frac{(2n)!}{2^n(n!)^2}$

Q.3 A) Attempt <u>any one</u>: 05

a) Let x_0 be in I, and let $\alpha_1, - - -, \alpha_n$ be any n constants. Prove that there is at most one solution $\emptyset$ of
$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + - - - + a_n(x)y = 0$
On I satisfying
$\emptyset(x_0) = \alpha_1, \emptyset'(x_0) = \alpha_2, - - --, \emptyset^{(n-1)}(x_0) = \alpha_n$

b) Find first four Legendre polynomials
$P_0(x), P_1(x), P_2(x) and\ P_3(x)$

B) Attempt <u>any one</u>: 05

c) Find all solutions of the equation for $x > 0$
$x^3y''' + 2x^2y'' - x\,y' + y = 0$

d) Compute the indicial polynomial and roots of indicial polynomial for the equation.
$x^2y'' + (x + x^2)y' - y = 0$

Q.4 Choose the correct alternative: 10

1) The equation $x^2y'' + a.xy' + by = 0$, (a, b constant) is
a) Legendre equation
b) Euler equation
c) Bessel equation
d) None of these

2) The solutions of the equation $x^2y'' + xy' + y = 0$, for $|x| > 0$, are
a) x, x^{-1}
b) $|x|^2, |x|^{-2}$
c) $|x|^i, |x|^{-i}$
d) $|x|^{2i}, |x|^{-2i}$

3) The singular point for the equation $a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + --- + a_n(x)y = 0$is the point x_0 for which
   a) $a_0(x_0) = 0$
   b) $a_0(x_0) \neq 0$
   c) $a_1(x_0) = 0$
   d) $a_1(x_0) \neq 0$

4) The solutions $\emptyset_1(x) = x$ and $\emptyset_2(x) = \frac{1}{x}$ of the equation $x^2y'' + xy' - y = 0$, for $x > 0$, are
   a) Linearly independent
   b) Linearly dependent
   c) Both (a) and (b)
   d) Neither (a) nor (b)

5) If $\emptyset_1, \emptyset_2$ are linearly independent solutions of equation $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I, also $\emptyset_1(x) \neq 0$ on I, then which of the following is wrong.
   a) $(\emptyset_1, \emptyset_2)(x) = \exp\left[-\int_{x_0}^{x} a_1(t)dt\right](\emptyset_1, \emptyset_2)(x_0)$
   b) $(\emptyset_1, \emptyset_2)(x) \neq 0$ for all x in I
   c) $\emptyset_2(x) = \emptyset_1(x)\int_{x_0}^{x} \frac{1}{[\emptyset_1(s)]^2} \exp\left[-\int_{x_0}^{s} a_1(t)dt\right] ds$
   d) All above (a), (b) and (c) are wrong.

OR

SUBJECT CODE NO:- B-2037
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-VI) Examination March/April 2018
Mathematics MAT - 605
3 Programming in C-II-605

[Time: 1:30 Hours] **[Max.Marks: 40]**

Please check whether you have got the right question paper.

N.B
1. All questions are compulsory.
2. Assume the data wherever not given with justification.
3. Figures to the right indicate full marks.

Q.1 A) Attempt <u>any one</u>: 05

a) Explain nesting of if … else statements and draw flowchart.
b) Discuss The? : Operator.

B) Attempt <u>any one</u>: 05

c) What is the output of the following program?

```
main ( )
{
     int m =5;
     if (m<3) printf (“% d”, m+1);
     else if (m<5) printf (“% d”, m+2);
     else if (m<7) printf (“% d”, m+3);
     else printf (“% d”, m+4);
}
```

d) Write a program to evaluate the square root for four numbers.

Q.2 A) Attempt <u>any one</u>: 05

a) What is jumping out of a loop? Explain in detail.
b) Explain in brief the additional features of for loop.

B) Attempt <u>any one</u>: 05

c) Write a program to print the multiplication table from 1×1 to 5×10 as shown below.

1	2	3	4	…	10
2	4	6	8	…	20
3	6	9	12	…	30
4	8	12	16	…	40
5	10	15	20	…	50

d) Using break statement, write a program to read a list of positive values and calculate their average.

Q.3 A) Attempt any one: 05

a) Write a note on multi-dimensional arrays.
b) Discuss searching and sorting operations in c-language.

B) Attempt any one: 05

c) What is the output of the following program?

```
main ( )
{
  chart string [ ] = "HELLO WORLD";
   int m;
  for (m =0; string [m]! = '\0'; m++)
          if ((m%2) = = 0)
     printf ("% c", string [m]);
}
```

d) Write a program to read the city code and the car code, one set after another from the terminal.

Q.4 Fill in the blanks: 10

1) A sorted list in array is called ----------.
2) The body of the loop may have ------ or ---------- statements.
3) The general form of initialization of arrays is ----------.
4) The initialization and increment sections are omitted in the ---------.
5) A counter-controlled loop is called ---------.

Total No. of Printed Pages:03

SUBJECT CODE NO:- B-2066
FACULTY OF SCIENCE
B.Sc. S.Y (Sem-IV) Examination March/April 2018
Mathematics MAT – 403
Mechanics-II

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all question.
ii) Figure to the right indicate full marks.
iii) Draw well labelled diagram whenever necessary.

Q.1 A) Attempt any one : 08

a) Find the radial and transverse components of velocity

b) If the sum of external forces acting on a system of particles be zero in any direction then prove that the total momentum of the system in that direction remains same during the motion.

B) Attempt any one : 07

c) A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with uniform angular velocity. Show that intrinsic equation of path is of the form $S = Ae^{\Psi} + B$

d) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h, show that the velocity of recoil is

$$\left(\frac{2m^2gh}{M(m+M)}\right)^{\frac{1}{2}}$$

Q.2 A) Attempt any One : 08

a) Find the equation of the parabola of the safety.

b) Find the differential equation of the path of a particle moving under a central force f(r) directed towards a fixed point 0 in a plane in polar form.

B) Attempt any One : 07

c) If the time of flight of a projectile over a horizontal range R is t and T seconds for two angles of projection α and β then prove that, $t^2 \cot \propto = T^2 \cot \beta$.

d) A particle describes the curve $r^n = A\cos n\theta + B\sin n\theta$ under the force F to the pole. Find the low of force.

Q.3 A) Attempt any One : 05

a) Express actual areal speed in terms of the angular speed.

b) If the velocity of a particle increases from $\overrightarrow{V_1}\ to\ \overrightarrow{V_2}$, then prove that the gain in the kinetic energy is half the scalar product of impulse and the sum of $\overrightarrow{V_1}\ and\ \overrightarrow{V_2}$,

B) Attempt any One: 05

c) A particle describes the curve $x = 5t, y = 3t^2$. Find the components of tangential and normal acceleration at $t = 2$ sec.

d) Find the work done by the force $\vec{F} = 2x\,\vec{\imath} + 2y\,\vec{\jmath}$ in a moving partial form P (1, 2) to Q(3,2).

Q.4 Choose the correct alternative and rewrite the sentence. 10

i) Actions and reactions are _______
- a) equal and opposite
- b) not equal
- c) equal but not opposite
- d) not equal but opposite

ii) Latus reaction of the parabola of safety is _______
- a) $\frac{2u}{9}$
- b) $\frac{2u}{g^2}$
- c) $\frac{2u^2}{g}$
- d) $\frac{2u^2}{g^2}$

iii) The path of projectile in vacuum is _______
- a) a plane curve
- b) a cycloid
- c) a circle
- d) a parabola

iv) The central orbit is _______
- a) a triangle
- b) a plane curve
- c) a parallelogram
- d) parabola

v) The radial component of acceleration is _______

a) $r\left(\frac{d\theta}{dt}\right)^2$

b) $r\frac{d\theta}{dt}$

c) $r\ \frac{d^2\theta}{dt^2}$

d) $r^2\left(\frac{d\theta}{dt}\right)^2$

Total No. of Printed Pages:03

SUBJECT CODE NO:- B-2147
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-V) Examination March/April 2018
Mathematics MAT – 501
Real Analysis - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one:

a) If $A_1, A_2, A_3 ------$ are countable sets, then prove that $\bigcup_{n=1}^{\infty} A_n$ is countable. 08

b) If $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} S_n = M$ then prove that L =M. 08

B) Attempt any one:

c) Prove that the set of all rational numbers in [0,1] is countable. 07

d) Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ is convergent. 07

Q.2 A) Attempt any one:

a) If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers. If $\lim_{n\to\infty} S_n = L$ and $\lim_{n\to\infty} t_n = M$ then prove that $\lim_{n\to\infty}(S_n - t_n) = L - M$ 08

b) If $\{S_n\}_{n=1}^{\infty}$ is Cauchy sequence of real numbers then prove that $\{S_n\}_{n=1}^{\infty}$ is convergent. 08

B) Attempt any one:

c) Using $\epsilon.\delta$ method, prove that $\lim_{n\to\infty} \frac{2n^3+5n}{4n^3+n^2} = 2$ 07

d) If $u = x^2 + y^2 + z^2$,V= $x + y + z$, $w = xy + yz + zx$. Show that the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ vanishes identically. 07

Q.3 A) Attempt any one:

a) If $o < x < 1$, then prove that $\sum_{n=0}^{\infty} x^n$ Converges to $\frac{1}{1-x}$. 05

b) If $\sum_{n=1}^{\infty} a_n$ is dominated by $\sum_{n=1}^{\infty} b_n$ where $\sum_{n=1}^{\infty} b_n$ converges absolutely, then prove that $\sum_{n=1}^{\infty} a_n$ converges absolutely. 05

B) Attempt any one:

c) Show that the series $\sum_{n=1}^{\infty} \frac{2n}{(n^2-4n+7)}$ is divergent. 05

d) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is convergent. 05

Q.4 Choose correct alternative: 10

i) If $B = \left\{\frac{1}{2}, \frac{3}{4}, - - - - - \frac{(2^n-1)}{2^n}, - - - -\right\}$
Then l.u.b x is
$x \in B$

a) 0
b) $1/2$
c) 1
d) None of these

ii) $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}}$ is
a) 1
b) 0
c) ∞
d) None of these

iii) If $\{S_n\}_{n=1}^{\infty}$ diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ converges to M then $\lim_{n\to\infty}\{S_n + t_n\}$ is
a) 0
b) M
c) ∞
d) $1/M$

iv) The sequence $\left\{\log\left(1/n\right)\right\}_{n=1}^{\infty}$ is

a) Diverges to minus infinity
b) Convergent
c) Diverges to plus infinity
d) None of these

v) The series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is

a) Convergent
b) Diverges to plus infinity
c) Diverges to minus infinity
d) None of these

Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2148
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-V) Examination March/April 2018
Mathematics MAT – 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) Attempt all questions.
2) Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) Define group and prove that cancellation laws holds in group.
b) If H is a nonempty finite subset of a group G and H is closed under multiplication then prove that H is a subgroup of G.

B) Attempt any one: 07

c) Prove that HK, where H and K are subgroups of G, is subgroup of G if and only if HK=KH.
d) If ϕ is an automorphism of G and a∈G is of order O(a)>1, then prove that O(ϕ (a)) = O(a).

Q.2 A) Attempt any one: 08

a) Prove that every finite integral domain is a field.
b) If f(x),g(x) are two non zero elements of F(x) then prove that
$$\deg\big(f(x).g(x)\big) = \deg f(x) + \deg g(x)$$

B) Attempt any one: 07

c) If D is an integral domain of finite characteristic then prove that the characteristic of D is prime.
d) If P is a prime number then prove that J_p , the ring of integer modulo P is a field.

Q.3 A) Attempt any one: 05

a) If H is a subgroup of G and N is a normal subgroup of G prove that $H \cap N$ is a normal subgroup of H.
b) If n is a positive integer and a is relatively prime to n then prove that
$$a^{\phi(n)} \equiv 1 \ (mod\ n)$$

B) Attempt any one: 05

c) If $a \in G,$ define $N(a) = \{x \in G | x.a = a.x\}$ then show that N(a) is a subgroup of G.
d) For all $a \in G,$ show that
$Ha = \{x \in G \mid a \equiv x \ (mod\ H)\}$

Q.4 Choose correct alternative. 10

i) If $O(H) = 4, O(K) = 5, O(H \cap K) = 2$ then $O(HK) =$ ------------
a) 4
b) 5
c) 2
d) 10

ii) In S_3 order of Ψ is -------------
a) 3
b) 2
c) 1
d) 6

iii) The group of integer modulo 5 has --------------- generators
a) 2
b) 4
c) 3
d) 5

iv) If every $x \in R$ satisfies $x^2 = x$ then R must be
a) Non commutative
b) Field
c) Commutative
d) Integral domain

v) If F is a field then F[x] is ------------
a) Integral domain
b) Field
c) Neither field nor Integral D
d) None of these.

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2163
FACULTY OF SCIENCE
B.Sc. F.Y (Sem-I) Examination March/April 2018
Mathematics MAT - 101
Differential Calculus

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all questions.
ii) Figures to the right indicate full marks.

Q.1 (A) Attempt any one: 08

(a) For $x, y \in R$, prove that $\cosh(x+y) = coshx\ coshy + sinhx\ sinhy$

(b) If $y = \sin(ax+b)$, prove that $\frac{d^n y}{dx^n} = a^n \sin(ax+b+n\frac{\pi}{2})$ and apply it to evaluate $\frac{d^5 y}{dx^5}$ if $y = Sin(3x+2)$

(B) Attempt any one: 07

(c) If $Y = Cos(msin^{-1}x)$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$

(d) If $y = a\cos(logx) + b\sin(\log x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

Q.2 (A) Attempt any one: 08

a) If $z = f(x,y)$is a homogeneous function of x, y of degree n, then prove that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z$

b) If a function f is
i) Continuous in closed interval [a, b] and
ii) Derivable in open interval (a,b), then prove that there exists at least one value $c \in (a,b)$ such that $\frac{f(b)-f(a)}{b-a} = f'(c)$

(B) Attempt any one: 07

c) Verify Rolle's theorem for the function $f(x) = \frac{sinx}{e^x}\ in\ [0,\pi]$

d) Verify that $\frac{\partial^2 y}{dxdy} = \frac{\partial^2 y}{\partial y\partial x}$
When $u = sin^{-1}(\frac{x}{y})$

Q.3(A) Attempt any one: 05

a) Prove that
grad$(\vec{f}.\vec{g})$ = $\vec{f} \times curl\, \vec{g} \times curl\, \vec{f} + (\vec{f}.\nabla)\vec{g} + (\vec{g}.\nabla)\vec{f}$

b) Prove that the function div $\vec{f}$ and curl $\vec{f}$ are point functions.

(B) Attempt any one: 05

c) Find grad ø if ø = $2x^2y^3 - 3y^2z^3$ at the point (1, -1,1)

d) If a function f is defined by $f(x) = \frac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, x \neq 0$

$= 0, \quad x = 0$

Show that f is continuous at $x = 0$.

Q.4 Choose the correct alternative. 10

i) Which of the following is not true?
(a) If f is a finitely derivable at C then it is also continuous at C
(b) $|x|$ is derivable at 0
(c) $f(x)$is derivable at $x = a\ if\ R[f'(a)] = L(f'(a)]$
(d) For $x \in R$, $\cosh(-x) = coshx$

ii) If $y = \log(\sin x), then\ Y_3 = \cdots \ldots \ldots$
(a) $\frac{2cosx}{sin^3x}$
(b) $\frac{2\ sinx}{cos^3x}$
(c) $2\ tanx$
(d) sin^3x

iii) For every $x \in R, e^x = \ldots\ldots$
(a) $x - \frac{x^2}{2!} + \frac{x^4}{4!} \ldots\ldots.$
(b) $1 + x + \frac{x^2}{2!} + \ldots\ldots.$
(c) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots.$
(d) None of these

iv) If $\vec{a}$ is a constant vector then div $(\vec{r} \times \vec{a}) =$.........

(a) 0
(b) 1
(c) 2
(d) 3

v) grad $(\vec{r}.\vec{a}) =$........., where $\vec{a}$ is a constant vector

(a) 0
(b) $\vec{a}$
(c) $2\vec{a}$
(d) 1

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2164
FACULTY OF SCIENCE
B.Sc. F.Y (Sem-I) Examination March/April 2018
Mathematics MAT – 102
Differential Equations

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N.B
i) Attempt all questions.
ii) Figures to the right indicate full marks.

Q.1 A) Attempt any one. 08

a) Prove that the necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is that
$$\frac{\partial M}{\partial Y} = \frac{\partial N}{\partial X}$$

b) Explain the method of solving the differential equation $\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}}$ +-------+$P_n y = x$, where P_1, P_2 ,------------------- Pn are constants and x is a function of x.

B) Attempt any one. 07

c) Solve the simultaneous equations.
$$\frac{dx}{dt} - 7x + y = 0 , \frac{dy}{dt} - 2x - 5y = 0$$

d) Solve $\frac{d^3y}{dx^3} + y = 3 + e^{-x} + 5e^{2x}$

Q.2 A) Attempt any one. 08

a) Explain the method of solving the differential equation
$(a + bx)^n \frac{d^n y}{dx^n} + (a + bx)^{n-1} \frac{d^{n-1}y}{dx^{n-1}}$ +--------+$P_{n-1}(a + bx) \frac{dy}{dx} + P_n y = F(x)$,

where P_1, P_2------P_n are constants.

b) Explain the method of solving the differential equation
$x^n \frac{d^n y}{dx^n} + P_1 x^{n-1} \frac{d^{n-1}y}{dx^{n-1}}$ + ---------+$P_{n-1}\, x.\frac{dy}{dx} + P_n y = x$,

where P_1, P_2 -------------P_n are constants and x is a function of x.

B) Attempt any one. 07

c) Solve $x^2\frac{d^2y}{dx^2}+7x\frac{dy}{dx}+5y=x^5$

d) Solve $\frac{d^2y}{dx^2}+4y=x\,sinx$

Q.3 A) Attempt any one. 05

a) With usual notation, prove that

$\frac{1}{f(D)}xv=\left\{x-\frac{1}{f(D)}f'(D)\right\}\frac{1}{f(D)}v$, Where v is any function of x.

b) Explain the method of solving the differential equation of the form

$\frac{d^2y}{dx^2}=f(y)$

B) Attempt any one. 05

c) Solve $\frac{dy}{dx}+\frac{2}{x}y=3x^2y^{4/3}$

d) Form the partial differential equation corresponding to $(x-h)^2+(y-k)^2+z^2=c^2$

Q.4 Choose correct alternative. 10

i) The general solution of the differential equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-54\,y=0\ is$ --------

a) $y=C_1e^{6x}+C_2e^{-9x}$

b) $y=c_1\cos(6x)+c_2\sin(-9x)$

c) $y=c_1e^{-6x}+C_2e^{9x}$

d) $y=c_1e^{-x}+c_2e^{-9x}$

ii) The integrating factor of the differential equation $\cos^2x\frac{dy}{dx}+y=\tan x$ is ----------------

a) $\tan x$
b) $\log(\tan x)$
c) $e^{\tan x}$
d) $\sec^2 x$

iii) The particular integral of differential equation $\frac{d^3y}{dx^3} + y = 5\,e^{-x}$ is -----------------

a) $5xe^{-x}$

b) $\frac{5}{3}xe^{-x}$

c) xe^{-x}

d) $3xe^{-x}$

iv) The partial differential equation obtained by eliminating constants a and b from $z = a(x + y) + b$ is -------------

a) $pq = 1$
b) $p^2 = q^2$
c) $p = q$
d) None of these

v) The partial differential equation corresponds to ---------------------.

a) Single independent variable
b) More than one independent variable
c) Single ordinary derivative
d) None of these

Total No. of Printed Pages:02

SUBJECT CODE NO:- B-2169
FACULTY OF SCIENCE
B.Sc. S.Y (Sem-III) Examination March/April 2018
Mathematics MAT - 301
Number Theory

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i. Attempt all questions.
ii. Figures to the right indicate full marks.

Q.1 A. Attempt any one. 08

a. If a, b, c be the integers then prove the following.
   i) If a|b and the b|c then a|c.
   ii) If a|b and a|c then $a|(bx + cy)$ for arbitrary integers x and y.

b. If a and b be integers, not both zero. Then prove that a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$.

B. Attempt any one. 07

c. By using division algorithm show that square of any integer is of the form 3k or $3k + 1$.

d. Using Euclidean Algorithm, find x and y if $\gcd(56, 72) = 56x + 72y$.

Q.2 A. Attempt any one 08

a. If P is prime, then prove that $(P-1)! \equiv -1(mod\ P)$

b. If a, b, c are arbitrary integers and $n > 1$ is fixed, then prove that.
   i) If $a \equiv b\ (mod\ n)$ and $b \equiv c(mod\ n)$ then $a \equiv c\ (mod\ n)$
   ii) If $a \equiv b\ (mod\ n)$ then $a^k \equiv b^k(mod\ n)$ where k is positive integer.

B. Attempt any one. 07

c. Solve the linear congruence $18x \equiv 30(mod\ 42)$.

d. By using Fermat's theorem, derive the following congruence equation $a^7 \equiv a(mod\ 42)$.

Q.3 A. Attempt any one. 05

a. If F is a multiplicative function and $F(n) = \sum_{d/n} f(d)$ then show that f is also multiplicative.

b. If P is prime and $k > 0$ then prove that $\emptyset(P^k) = P^k(1 - \frac{1}{P})$

B. Attempt any one. 05

c. Calculate $\emptyset(360)$.

d. Find the remainder when 2^{50} is divided by 7.

Q.4 Choose the correct alternative. 10

i. If P is prime number, then

a) $\mu(P) = 0$

b) $\mu(P) = 1$

c) $\mu(P) = -1$

d) None of these

ii. If P is prime, then

a) $(P-1)! \equiv 0 \ (mod\ P)$

b) $(P-1)! + 1 \equiv 0 \ (mod\ P)$

c) $(P-1)! \equiv 2 \ (mod\ P)$

d) None of these

iii. The linear congruence $ax \equiv b(mod\ n)$ has unique solution. Modulo n, if

a) $\gcd(a, b) = 1$

b) $\gcd(b, n) = 1$

c) $\gcd(a, n) = 1$

d) None of these

iv. If $a = 12$ and $b = 30$ then $lcm(a,\ b)$ is

a) 30

b) 120

c) 180

d) 60

v. Value of $\sigma(12)$ is

a) 28

b) 12

c) 18

d) 22

Total No. of Printed Pages:3

SUBJECT CODE NO:- B-2170
FACULTY OF SCIENCE
B.Sc. S.Y (Sem-III) Examination March/April 2018
Mathematics MAT – 302
Integral Transforms

[Time: 1:30 Hours] **[Max.Marks: 50]**

Please check whether you have got the right question paper.

N.B
1. Attempt all questions.
2. Figures to the right indicate full marks.

Q.1 A) Attempt any one: 08

a) Prove that:
$\beta(l,m) = \frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$,
where l, m are integer

b) Prove that:
$\Gamma(n) = (n-1)!$,
where n is a positive integer

B) Attempt any one: 07

c) Show that
$$\int_0^\infty \frac{\sin bz}{z}\, dz = \frac{\pi}{2}$$

d) Show that
$$2^n\Gamma\left(n+\frac{1}{2}\right) = 1.3.5.---(2n-1).\sqrt{\pi}$$

Q.2 A) Attempt any one: 08

a) If $L\{F(t)\} = f(s)$,
Then prove that
$$L\left\{\int_o^t F(u)du\right\} = \frac{1}{s}f(s)$$

b) If $L^{-1}\{f(s)\} = F(t), L^{-1}\{g(s)\} = G(t)$,
Then prove that

$$L^{-1}\{f(s).g(s)\} = \int_{o}^{t} F(u)G(t-u)du$$

B) Attempt <u>any one</u>: 07

c) Find

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)\,(s^2+b^2)}\right\}, a \neq b$$

d) Show that

$$L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\} = \frac{1}{2a^3}\{\sin at - at\cos at\}$$

Q.3 A) Attempt <u>any one</u>: 05

a) If $F\{F(x)\} = f(s)$ then prove that

$$F\{F(x)\cos ax\} = \frac{1}{2}f(s-a) + \frac{1}{2}f(s+a)$$

b) If $f(s)$ is the Fourier transform of $F(x)$, then prove that $\frac{1}{a}f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(ax)$

B) Attempt <u>any one</u>: 05

c) Find the Fourier transform of

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

d) Solve
$(D+2)^2 y = 4e^{-2t}$,
with $y(0) = -1, y'(0) = 4$
using Laplace transform

Q.4 Choose the correct alternative: 10

1) The value of $\Gamma(1)$ is
a) 0
b) $\frac{1}{2}$
c) 1
d) -1

2) The value of the integral $\int_0^1 x^4(1-x)^3 dx$ is

a) $\frac{1}{140}$

b) $\frac{1}{280}$

c) $\frac{3}{280}$

d) $\frac{3}{140}$

3) If $f(t) = 1$, then the value of $L\{f(t)\}$ is

a) $\frac{1}{s}$

b) $\frac{1}{s^4}$

c) $\frac{1}{s^2}$

d) $\frac{1}{s-1}$

4) The value of $L^{-1}\left\{\frac{1}{s^2}\right\}$ is equal to

a) $\frac{1}{t}$

b) t^2

c) $\frac{2}{t}$

d) t

5) The value of $\Gamma(6)$ is equal to

a) 60
b) 240
c) 120
d) 125

Total No. of Printed Pages:03

SUBJECT CODE NO: B-2173
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-V) Examination March/April 2018
Mathematics MAT-503
1) Mathematical Statistics - I

[Time: 1:30 Hours] **[Max.Marks: 50]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Calculator is allowed.

Q.1 (A) Attempt any one:

a) Explain frequency distribution with suitable example. 08

b) State and prove the formula for the mean of composite series. 08

(B) Attempt any one:

c) Find the mean and variance of first n – natural numbers. 07

d) Find the mode of the following frequency distribution: 07

Class intervals	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	14	23	27	21	15

Q.2 (A) Attempt any one:

a) Prove that variance is not independent of change of scale. 08

b) Find the relation between moments about mean in terms of moments about any point. 08

(B) Attempt any one:

c) Find the average deviation from mean of the following frequency distribution: 07

Class intervals	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	8	10	12	9	5

d) Find the standard deviation of the set of numbers 3, 4, 9, 11, 13, 6, 8 and 10. 07

Q.3 (A) Attempt <u>any one</u>:

a) Define probability function for a fixed B with $P(B) > 0$, show that $P(A|B)$ is a probability function. 05

b) With usual notations, prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. 05

(B) Attempt <u>any one</u>:

c) Write down the sample space when: 05
   i) A coin is tossed 03 times,
   ii) Two dice are thrown simultaneously.
   iii) Four cards are drawn from the Pack of playing cards at random.

d) Let x be a continuous random variable with p.d.f. 05

$$f(x) = ax\,, \quad 0 \le x \le 1$$
$$= a\,, \quad 1 \le x \le 2$$
$$= -ax + 3a\,, \quad 2 \le x \le 3\,,$$
$$= 0\,, \quad elsewhere$$

Find the value of the constant 'a'.

Q.4 Choose the correct alternative from the following: 10

i) The mean of a set of 7 observations is 10 and the mean of a set of 3 observations is 5, then mean of the combined set is
   a) 15
   b) 10
   c) 8.5
   d) 7.5

ii) The probability of certain event and impossible event is
   a) 1 and 0
   b) 10 and 0
   c) 0 and 1
   d) Does not exist.

iii) The minimum value of mean square deviation is
   a) Mean
   b) Median
   c) Variance
   d) Zero

iv) If $f(x) = kxe^{-x}, (0 \leq x < \infty)$ be a continuous distribution, then the value of K is

a) 0
b) 1
c) 10
d) 5

v) The probability of drawing one white ball from a bag containing 6 red, 8 black, 10 yellow and 5 green balls is

a) 1
b) $\frac{1}{29}$
c) 29
d) Zero

OR

Total No. of Printed Pages:03

SUBJECT CODE NO: B-2173
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-V) Examination March/April 2018
Mathematics MAT - 504
2) Ordinary Differential Equation - I

[Time: 1:30 Hours] **[Max.Marks: 50]**

Please check whether you have got the right question paper.

N.B 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 (A) Attempt any one:

a) Consider the equation 08
$y' + ay = 0$
Where **a** is complex constant. If C is any complex number, then prove that the function ϕ defined by
$\phi(x) = ce^{-ax}$
is solution of this equation and also prove that every solution has this form.

b) If P is a polynomial of degree $n \geq 1$, with leading coefficient 1 (the coefficient of z^n), and r is root of p. then prove that: 08
$$p(z) = (z - r)q(z)$$
Where q is a polynomial of degree $n - 1$, with leading coefficient 1.

(B) Attempt any one:

c) Find all solutions of 07
$y' + 2xy = x$

d) Find the three cube roots of 4i. 07

Q.2 (A) Attempt any One:

a) Prove that for any real x_0 and constants α, β, there exists a solution ϕ of the initial value problem: 08
$L(y) = y" + a_1 y' + a_2 y = 0$
$y(x_0) = \alpha, y'(x_0) = \beta.$
$on - \infty < x < \infty$

b) If ϕ_1, ϕ_2 are any two solutions of 08
$L(y) = y" + a_1 y' + a_2 y = 0$
On an interval I and x_0 is any point in I. then prove that ϕ_1, ϕ_2 are linearly independent on I if and only if.
$W(\phi_1, \phi_2)(x_0) \neq 0$.

(B) Attempt <u>any one:</u>

c) Find all solutions of 07
$y" - y' - 2y = e^{-x}$

d) Suppose that ϕ is a function having a continuous derivative on $0 \leq x < \infty$, such that 07
$\phi'(x) + 2\phi(x) \leq 1$. For all such x, and $\phi(0) = 0$.
Show that $\phi(x) < \frac{1}{2}\ for\ x \geq 0$.

Q.3 (A) Attempt any One:

a) If ϕ_1, ϕ_2 are two solutions of 05
$L(y) = y" + a_1 y' + a_2 y = 0$
On an interval I containing a point x_0, then prove that
$W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$.

b) Solve the equation 05
$Ly' + Ry = E$
Where L, R, E are positive constants.

(B) Attempt any one:

c) If for all real x 05

$$f(x) = x + ix^2 \ , \qquad g(x) = \frac{x^2}{2}$$

Then find:
i) The function F given by $F(x) = f(g(x))$
ii) $F'(x)$

d) If the functions 05
$\phi_1(x) = sinx \ , \ \phi_2(x) = e^{ix}$
Exist for $-\infty < x < \infty$. Determine whether they are linearly dependent or independent there.

Q.4 Choose the correct alternative: 10

1. "If P is a polynomial such that $\deg p \geq 1$, then p has at least one root" this theorem is known as
   a) Fundamental theorem of algebra
   b) Fundamental theorem of Calculus.
   c) Fundamental theorem of Number theory
   d) None of these

2. The function ϕ is called solution of $y' = f(x, y)$ where $y \in S$ if
   a) $\phi(x)$ is in S.
   b) $\phi'(x) = f(x, \phi(x))$
   c) Both (a) and (b)
   d) None of these

3. The solution ϕ of $y' + ay = b(x)$ is given by
   a) $\phi(x) = e^{-ax} \int_{x_0}^{x} e^{at} b(t) dt$
   b) $\phi(x) = e^{-ax} \int_{x_0}^{x} e^{at} b(t) dt + ce^{-ax}$
   c) $\phi(x) = ce^{-ax}$
   d) None of these

4. If ϕ_1, ϕ_2 are any two solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ and c_1, c_2 are any two constants then the function
   $\phi = c_1\phi_1 + c_2\phi_2$ is ....
   a) Not a solution of $L(y) = 0$
   b) A solution of $L(y) = 0$
   c) Characteristic polynomial of $L(y) = 0$
   d) None of these

5. The functions $\phi_1(x) = x$, $\phi_2(x) = |x|$ are
   a) Linearly independent
   b) Linearly dependent
   c) Both (a) and (b)
   d) None of these

OR

Total No. of Printed Pages:02

SUBJECT CODE NO: B-2173
FACULTY OF SCIENCE
B.Sc. T.Y (Sem-V) Examination March/April 2018
Mathematics MAT- 505
3) Programming in C - I

[Time: 1:30 Hours] **[Max.Marks: 40]**

Please check whether you have got the right question paper.

N.B
1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Assume the data wherever not given with justification.

Q.1 (A) Attempt any one: 05

a) Draw a flowchart of process of compiling and running a C program.

b) Write the rules for identifiers in C language.

(B) Attempt any one: 05

c) Write a program in C to add two numbers.

d) Write a C program to convert the given temperature in Fahrenheit to Celsius.

Q.2 (A) Attempt any one: 05

a) Discuss integer data types in C language with example.

b) Explain logical operators with example.

(B) Attempt any one: 05

c) Write a C program to convert a given number of days into months and days using integer arithmetic.

d) What is output of the following program?

```
main( )
{
    char x;
    int  y;
    x = 100;
    y = 125;
    printf ("%c \n", x);
    printf("%c \n", y);
    printf("%d \n", x);
}
```

Q.3 (A) Attempt <u>any one:</u> 05

a) What care should be taken while using a scanf statement?

b) Write any ten mathematical functions used in C language and their meaning.

(B) Attempt <u>any one:</u> 05

c) Write a program to read character using getchar function.

d) Write a C program using a cast to evaluate the equation

$$sum = \sum_{i-1}^{n} \left(\frac{1}{i}\right)$$

Q.4 Fill in the blanks: 10

i) The variables used in more than one function are called ________.

ii) The backslash character constant $'\backslash t'$ means _________.

iii) The complement of relational operator $<= is$ ____________.

iv) Identifiers must not contain _______.

v) The _______ header file contains mathematical functions.

SUBJECT CODE NO: B-2204
FACULTY OF SCIENCE
B.Sc. S.Y (Sem-III) Examination March/April 2018
Mathematics MAT - 303
Mechanics-I

[Time: 1:30 Hours] **[Max.Marks:50]**

Please check whether you have got the right question paper.

N.B
i. Attempt all questions.
ii. Figure to the right indicates full marks

Q.1 A) Attempt any one 08

a) State and prove lami's theorem

b) Determine the magnitude and direction of the resultant $\vec{R}$ of the two forces $\vec{P}$ and $\vec{Q}$ acting at an angle Q

B) Attempt any one 07

c) Find the magnitude of the forces such that if they act at right angles, their resultant is $\sqrt{10}$ kg. but if they act at 60^0 their resultant is $\sqrt{13}$ kg.

d) A body of weight 52kg is suspended by two strings of the lengths 5m and 12m attached to the points in the same horizontal line whose distance apart is 13m. find the tensions of the strings.

Q.2 A) Attempt any one 08

a) Prove that the necessary and sufficient condition that a given system of forces acting upon a rigid body is in equilibrium is that the force sum and moment sum must separately vanish.

b) Define couple, prove that the vector moment of the resultant couple of two couples acting upon a rigid body is the sum of the vector moments of the given couples.

B) Attempt any one 07

c) Define center of gravity. Determine the center of gravity of uniform rod.

d) The following forces act at a point
i) 20kg inclined at 30° towards north to east
ii) 25kg towards north
iii) 30 kg towards north west and
iv) 35kg inclined at 40° towards south to west
Find the magnitude and direction of the resultant force

Q.3 A) Attempt <u>any one</u> 05

a) Prove that if three like or unlike parallel forces be in equilibrium the magnitude of each force varies as the distance between the other two.

b) Prove that the C.G. of a uniform triangular lamina is the same as that of three equal particles placed at the vertices of the triangle

B) Attempt <u>any one</u> 05

c) Find the smaller force, when the two forces act at an angle of 120° the greater force is 30kg and resultant is perpendicular to smaller one

d) Find the vector moment of a force $\overline{F} = \overline{j} + 2\overline{j} + 3\overline{k}$ acting at a point (-1, 2, 3) about the point (2, 3, 5)

Q.4 Choose the correct alternative 10

1) Two forces acting at a point of rigid body are in equilibrium if they are__________
   a) Equal in magnitude and in same direction
   b) Equal in magnitude and opposite in direction
   c) Opposite in direction
   d) Equal in magnitude

2) If $\vec{P}$ and $\overline{Q}$ are resolved parts of a force $\overline{R}$ then angle between $\overline{P}$ and $\overline{Q}$ is___________
   a) 0°
   b) 90°
   c) 180°
   d) 360°

3) If two forces of magnitude P each acting at an angle θ then the magnitude R of their resultant force is given by________
   a) $R = 2Pcos2\theta$
   b) $R = 2Psin2\theta$
   c) $R = 2Pcos\frac{\theta}{2}$
   d) $R = 2Psin\frac{\theta}{2}$

4) Centroid of the weighted point __________
   a) Does not exist
   b) Exists and is unique
   c) Exists but is not unique
   d) None of above

5) The effect of the couple acting on the body produce_______
   a) Only a motion of rotation
   b) Only a motion of translation
   c) Both a & b
   d) None of above