Total No. of Printed Pages: 2

SUBJECT CODE NO: - Y-2039
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem-I)
Examination March / April - 2023
Mathematics MAT - 101 Differential Calculus

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) Attempt all questions.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one: 08

a) Show that f'(c), is the tangent of the angle which the tangent line to the curve y= f(x) at the Point P[c, f(c)] makes with x-axis.

b) If $y = e^{ax} \sin(bx + c)$, then show that $\frac{d^n y}{dx^n} = (a^2 + b^2)^{\frac{n}{2}} e^{ax} \sin(bx + c + n \tan^{-1}(\frac{b}{a}))$

B) Attempt any one: 07

c) If $y = \frac{x+1}{x^2-4}$; then find $\frac{d^n y}{dx^n}$

d) Find the value of the n[th] derivative of $y = e^m \sin^{-1} x$ for $x = 0$.

Q2 A) Attempt any one: 08

a) State and prove Cauchy's mean value theorem.

b) If z = f(x, y) be a homogeneous function of x, y of degree n then prove that
$x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial Y} = nz,$
$\forall$ x, y $\in$ the domain of the function.

B) Attempt any one: 07

c) Expand $2x^3 + 7x^2 + x - 6$ in Powers of $(x-2)$.

d) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $x \neq y$, then show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u \cos u$$

Q3 A) Attempt any one: 05

a) prove that:
$\operatorname{curl}(\vec{f} \times \vec{g}) = \vec{f} \operatorname{div} \vec{g} - \vec{g} \operatorname{div} \vec{f} + (\vec{g}.\nabla)\vec{f} - (\vec{f}.\nabla)\vec{g}$.

b) Prove that:
$\operatorname{grad}(\phi\psi) = \phi \operatorname{grad} \psi + \psi \operatorname{grad} \phi$.

B) Attempt any one: 05

c) Find grad $log|\vec{r}|$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

d) Find div $\vec{f}$ and curl $\vec{f}$, where $\vec{f}$ = grad $(axy^2 + byz + cz^2x^3)$

Q4 Choose the correct alternative. 10

i) If $y = x|x|$, then value of $\frac{dy}{dx}$ at the origin= ______.

a) 1 b) x c) 0 d) 2x

ii) If $y = \log(ax + b)$, then $\frac{d^n y}{dx^n} =$ _____

a) $\frac{(-1)^{n-1}(n-1)!a^n}{(ax+b)^n}$ b) $\frac{(-1)^n n!a^n}{(ax+b)^{n+1}}$ c) $\frac{(-1)^n(n-1)!a^{n-1}}{(ax+b)^n}$ d) $\frac{(-1)^{n-1}(n-1)!a^{n+1}}{(ax+b)^n}$

iii) $\log(1 + x) =$________.

a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} +$ _____ $+ (-1)^{\frac{n}{2}} \frac{x^n}{n!} +$ ______

b) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} +$ ____ $+ \frac{(-1)^{n-1}x^n}{n} +$ _____

c) $1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^9}{9!} +$ _____ $+ \frac{x^{3n}}{(3n)!} +$ ______

d) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} +$ ____ $+ \frac{x^n}{n} +$ ____

iv) If f(x) = |x|, x ϵ [~1,1]. then f(x)______

a) Satisfy conditions of Lagrange's mean value theorem.

b) Does not satisfy conditions of Rolle's theorem.

c) Satisfy conditions of Rolle's theorem.

d) Satisfy conditions of Cauchy's mean value theorem.

v) If ψ is a constant, then grad $\psi =$ _____

a) 0 b) 1 c) -1 d) $-\psi$

Total No. of Printed Pages: 02

SUBJECT CODE NO: Y- 2040
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem-I)
Examination March / April - 2023
Mathematics MAT - 102 (Differential Equations)

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) Attempt all questions.
2) Figures to the right indicates full marks.

Q1 A) Attempt any one. 08

a) Explain the method of solving the differential equation $\frac{dy}{dx} + py = Qy^n$, where P and Q are functions of x.

b) Explain the method of solving the differential equation.
$\frac{d^n y}{dx^n} + P_1 \frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_n y = X$,where $P_1, P_2, \ldots\ldots P_n$ are constants and X is a function of x

B) Attempt any one 07

c) Solve $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos(2x)$

d) Solve $x^2 \frac{d^2y}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$

Q2 A) Attempt any one 08

a) Explain the method of solving the differential equation
$x^2 \frac{d^n y}{dx^n} + P_1 \ x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + P_{n-1} x \frac{dy}{dx} + P_n Y = X$,where $P_1, P_2, \ldots \ldots, P_n$ constants and X is a function of x.

b) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 3e^{5/2x}$

B) Attempt any one 07

c) Solve $(2x-1)^3 \frac{d^3y}{dx^3} + (2x-1)\frac{dy}{dx} - 2y = 0$

d) Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2 = 2\log x$

Q3 A) Attempt any one 05

a) Explain the method of solution of bimultaneous differential equation
$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$ where P,Q, R use functions of x,y,z

b) With usual notation, prove that
$\frac{1}{f(D)}(xv) = \left\{x - \frac{1}{f(D)}f^1(D)\right\}\frac{1}{f(D)}V,$
Where V is any function of x

B) Attempt any one 05

c) Solve $(2ax + by + g)dx + (2cy + bx + e)dy = 0$

d) Form the partial differential equation by eliminating the arbitrary function from $z = e^{ny}\phi(x - y)$

Q4 Choose the correct alternative 10

i. The integrating factor of differential equation $x\frac{dy}{dx} - ay = x + a\ is \ldots \ldots$
a) x^a b) $\frac{1}{x^a}$ c) $\frac{-a}{x}$ d) x^2

ii. The partial differential equation correspond to __________
a) Single independent variable
b) More than one independent variable
c) Single ordinary derivative
d) None of these

iii. The general solution of the differential equation
$\frac{d^n y}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + P_n y = X$ is
a) y=C.F +P.I
b) y=C.F –P.I
c) y=complementary function
d) none of these

iv. The partial differential equation obtained by eliminating constants a and b from $z = a(x + y) + b$ is ___________
a) pq =1 b) p=q c) $P^2 = q^2$ d) none of these

v. The particular integral of the differential equation
$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 2e^{2x}$ is _________
a) $\frac{2}{9}e^{2x}$ b) $\frac{1}{9}e^{2x}$ c) $2e^{2x}$ d) e^{2x}

Total No. of Printed Pages: 2

SUBJECT CODE NO: - Y-2054
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem-II)
Examination March / April - 2023
Mathematics MAT - 201 (Integral Calculas)

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) Attempt all questions.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one. 08

a) Obtain a reduction formula for $\int x^n e^{-x} dx$ and hence show that the improper integral $\int_0^\infty x^n e^{-x} dx = n!$ Where n is any positive integer.

b) Evaluate the definate integral

$$\int_0^{\pi/2} Sin^n x\, dx$$

where n is a positive integer

B) Attempt any one. 07

c) Evaluate $\int \frac{x^5 dx}{x^3-2x^2-5x+6}$

d) Evaluate $\int \frac{(x^3+2)}{(x-1)(x-2)^3} dx$

Q2 A) Attempt any one. 08

a) Evaluate $\int_a^b \cos h\, 2x\, dx$ as the limit of sum.

b) Find the area of the region lying x-axis and included between the circle $x^2 + y^2 - 2ax$ and the parabola $y^2 = ax$.

B) Attempt any one 07

c) Find the length of the arc of the curve $y = \log \tan h \left(\frac{x}{2}\right)$ from $x = 1\ to\ x = 2$

d) Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + sin\theta), y = a(1 + cos\theta)$ about $x - axis.$

Q3 A) Attempt any one 05

a) If $\vec{F}$ is any continuously differentiable vector point function ans S is a surface bounded by curve C, then prove that $\int_c \vec{F}.d\vec{r} = \int_s curl\,\vec{F}.\vec{n}\,ds$, where the unit normal vector $\vec{n}$ at any point of s is drawn in the sense in which a right handed screw would move when rotated in the sense of description of C.

b) Prove that $\int_S \vec{n} \times (\vec{a} \times \vec{r}) ds = 2aV$.

B) Attempt any one. 05

c) Show that $\frac{1}{3}\int_S \vec{r}.d\vec{a} = V$
where V is the volume enclosed by the surface S.

d) Evaluate $\int_C \vec{F}.d\vec{r}$ where
$\vec{F} = (2x^2 + y^2)\,\vec{i} + (3y - 4x)\,\vec{j}$
around the triangle ABC whose vertices are A(0,0) , B(2,0) , C(2,1).

Q4 Choose the correct alternatives. 10

1) $\int \frac{dx}{(2x-3)^3} =$
a) $\frac{-1}{4(2x-3)^2}$ b) $\frac{1}{4(2x-3)^2}$ c) $\frac{1}{(2x-3)^2}$ d) $\frac{-1}{(2x-3)^2}$

2) $\int_0^{\pi/2} \sin^9 x\, dx$
a) $\frac{315}{128}$ b) $\frac{-128}{315}$ c) $\frac{128}{315}$ d) 0

3) Perimeter of the cardioid $r = a(1 + \cos\theta)$is _________
a) 2a b) 4a c)6a d)8a

4) The volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the initial line is ______
a) $\frac{\pi ab^2}{3}$ b) $3\pi ab^2$ c) πab^2 d) $\frac{4}{3}\pi ab^2$

5) If C be a closed curve then $\oint \vec{r}.d\vec{r} =$ _____
a) r b) r^2 c) $\frac{1}{r}$ d) 0

Total No. of Printed Pages: 03

SUBJECT CODE NO: - Y-2055
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem-II)
Examination March / April - 2023
Mathematics MAT - 202 (Geometry)

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) Attempt all questions.
2) Figure to the right indicate full marks.

Q1 A) Attempt any one. 08

a) Prove that every equation of the first degree in x, y, z represents a plane.

b) Find the equations of the line passing through a given point $A(x_1, y_1, z_1)$ and having direction cosines l, m, n.

B) Attempt any one. 07

c) Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$

d) Find two points on the line $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z-5}{2}$ on either side of (2, -3, -5) and at a distance 3 from it.

Q2 A) Attempt any one. 08

a) Find the length of the perpendicular from a given point $P\ (x_1, y_1, z_1)$ *to a given line* $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-r}{n}$

b) Prove that a plane section of a sphere is a circle.

B) Attempt any one. 07

c) Find the image of the point P(1, 3, 4) in the plane $2x - y + z + 3 = 0$

d) Find the equation of the sphere passing through the origin and the points (1, 0, 0) , (0, 2, 0) and (0, 0, 3)

Q3 A) Attempt any one. 05

a) Find the equation of the right circular cylinder whose axis is the line $\frac{x-\propto}{l}=\frac{y-\beta}{m}=\frac{z-r}{n}$ and whose radius is r.

b) Find the points of intersection of the line $\frac{x-\propto}{l}=\frac{y-\beta}{m}=\frac{z-r}{n}$ with the central conicoid $ax^2+by^2+cz^2=1$

B) Attempt any one.

c) Show that the distances between the parallel planes $2x-2y+2+3=0$ and $4x-4y+2z+5=0$ is $^1/_6$ 05

d) Find the equation of the right circular cylinder whose radius is 2 and axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$

Q4 Choose the correct alternatives and fill in the blanks. 10

1) Two planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ are parallel if ______.
   a) $a_1\,a_2+b_1\,b_2+c_1\,c_2=0$
   b) $\frac{a_1}{a_2}+\frac{b_1}{b_2}+\frac{c_1}{c_2}=0$
   c) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$
   d) $None\ of\ these$

2) The equations to the x-axis are _______
   a) $\frac{x}{0}=\frac{y}{1}=\frac{z}{1}$
   b) $\frac{x}{2}=\frac{y}{0}=\frac{z}{1}$
   c) $\frac{x}{1}=\frac{y}{1}=\frac{z}{0}$
   d) $\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$

3) The line $\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}$ and the plane $4x+5y+3z-5=0$ intersect at a point _________
   a) (3, 1, -2)
   b) (3, -2, 1)
   c) (2, -1, 3)
   d) (-1, -2, -3)

4) The radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$ is ______
   a) 49
   b) 5
   c) -7
   d) $\sqrt{7}$

5) The locus of the points of intersection of two spheres is a _____
   a) Circle
   b) Plane
   c) Conicoid
   d) Cylinder

Total No. of Printed Pages: 02

SUBJECT CODE NO: - YY-2349
FACULTY OF SCIENCE AND TECHNOLOGY
B. Sc. F.Y (Sem-II)
Examination March / April - 2023
Mathematics Paper -III
Number Theory

[Time: 1:30 Hours] **[Max. Marks: 40]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A. Attempt any one:

a. If $m > 0$, then prove that $[ma, mb] = m[a, b]$ 05

b. If plab , P being a prime, then prove that p/a or p/b. 05

B. Attempt any one:

c. Find the greatest common divisor of 7469 and 2464. 05

d. Prove that $n^3 - n$ is divisible by 6. 05

Q2 A. Attempt any one:

a. If $ax \equiv ay\ (mod\ m)$ and $(a, m) = 1$, then prove that $x \equiv y(mod\ m)$. 05

b. If P is a prime, then prove that $(p-1)! \equiv -1(mod\ p)$ 05

B. Attempt any one

c. Prove that if P is a prime and $a^2 \equiv b^2(mod\ p)$, then prove that $p \mid a+b\ \ or\ p|a-b$. 05

d. Find all integers that satisfy simultaneously : $x \equiv 2(mod\ 3)$, $x \equiv 3(mod\ 5), x \equiv 5(mod\ 2)$ 05

Q3 A. Attempt any one

a. If x is real number, then prove that $[x] \leq x < [x] + 1, x - 1 < [x] \leq x, 0 \leq x - [x] < 1$. 05

b. For every positive integer n, $\sum_{d/n} \Phi\ (d) = n$ 05

B. Attempt any one

c. Prove that $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$, n is positive integer. 05

d. Find all integers x and y such that $147x + 258y = 369$ 05

Q4 Choose the correct alternative and rewrite the sentences: 10

1. If a and b are integers with a > 0 then there exist unique integers q and r such that $b = qa + r, whree$
   a. $o \leq r \leq a$ b. $o \leq r \leq a$ c. $o < r \leq a$ d. $o < r < a$

2. The product of any three consecutive integers is divisible by ____
   a. 4 b. 5 c. 6 d. 7

3. If m is positive integer then $a \equiv b\ (mod\ m)$ if and only if ______
   a. $m/a + b$ b. $m/a - b$ c. m/ab d. $m/ma + b$

4. If d(n) denotes the _____ positive divisors of n, then d(12)= _______
   a. 28 b. 24 c. 12 d. 6

5. If μ is a Mobious function then $\mu(8) =$ ____
   a.-1 b. 1 c. 0 d. 8

Total No. of Printed Pages: 02

SUBJECT CODE NO: - YY-2350
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. F.Y (Sem- II)
Examination March / April - 2023
Mathematics Paper -IV Integral Calculus

[Time: 1:30 Hours] **[Max. Marks: 40]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A. Attempt any one. 05

a) Obtain a reduction formula for $\int \cos^n x \, dx$, where n is any integer.

b) Evaluate : $\int \frac{1}{x^2-x-6} \, dx$.

B. Attempt any one. 05

c) Evaluate : $\int \frac{dx}{\sin^3 x \ \cos^5 x}$

d) Evaluate : $\int \frac{dx}{sinzx-sinx}$

Q2 A. Attempt any one. 05

a) Evaluate $\int_a^b x^2 \, dx$ as the limit of a sum.

b) Show that the intrinsic equation of the semi-cubical parabola

$3ay^2 = 2x^3 \ \ is \qquad S = \frac{4}{9} a(\sec^3 \psi - 1)$

B. Attempt any one. 05

c) Find the whole length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$

d) The part of the parabola $y^2 = 4ax$ cut by the latus rectum revolves about the fangent of the Vertex. Find the volume of the reel thus generated.

Q3 A. Attempt any one. 05

a) Prove that, the field $\overline{F}$ is conservative over a region if and only if $\oint \overline{F} . d\overline{r} = 0$ along any closed curve in the region.

b) Prove that

$\int_S \overline{r}.\overline{n} \, ds = 3V$,where S is closed surface, and V is volume enclosed.

B. Attempt any one: 05

c) Evaluate $\int_c \bar{F}.d\bar{r}$, where $\bar{F} = x^2y^2\,\hat{\imath} + y\hat{\jmath}$, and the curve C is $y^2 = 4x$ in the xy-plane from (0,0) to (4,4)

d) Evaluate $\int_c \bar{F}.d\bar{r}$ by stoke's theorem where $\bar{F} = y^2\hat{\imath} + x^2\hat{\jmath} - (x+z)\hat{k}$ and c is the boundry of the friangle whose vertices are (0,0,0) , (1,0,0) and (1,1,0)

Q4 Choose the correct alternative and rewrite the sentence. 10

1) The value of $\int_0^{\pi/2} \sin^3 x\, dx =$ ………..
   a) $\frac{3}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{5}{3}$

2) Rectification is the process of determining ……………………
   a) The area under curve
   b) The arc length of plane curve
   c) Double integral
   d) Multiple integral

3) $x = a(\theta - sin\theta)$, $y = a\,(1 - cos\theta)$ is an equation of ……………
   a) Cycloid
   b) Cardiod
   c) Astroid
   d) None of these

4) The volume of the solid generated by the revolution about the x-axis, of the area bounded by the curve y=f(x), the ordinates at x=a, x=b, and the x-axis, is …….
   a) $\int_a^b x^2\, dx$ b) $\pi \int_a^b x^2\, dy$ c) $\int_a^b y\, dx$ d) $\pi \int_a^b y^2\, dx$

5) If the circulation of a vector point function $\bar{F}$ along any closed curve in a region is zero, then $\bar{F}$ is said to be …………
   a) Irrational b) Rotational c) Solenoidal d) None of these

Total No. of Printed Pages: 02

SUBJECT CODE NO: - Y-2050
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-III)
Examination March / April - 2023
Mathematics MAT - 301 Number Theory

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
i) All questions are compulsory.
ii) Figures to the right indicate full marks.

Q1 (a) Attempt any **one** of the following: 08

i. Given two integers a and b, with b > 0, then prove that there exist unique integers q and r such that
$$a = qb + r, 0 \le r \le b.$$

ii. If $k > 0$, then prove that $gcd\ (ka, kb) = k\ gcd\ (a, b)$.

(b) Attempt any **one** of the following: 07

i. By using the Euclidean algorithm, find the values of integers x and y satisfying $gcd(119, 272) = 119x + 272y$.

ii. If a and b are both odd integers, the prove that 16| $a^4+b^4- 2$.

Q2 a) Attempt any **one** of the following: 08

i. If n> 1 is a fixed integer and a, b, c, d are arbitrary integers, then prove that
$\alpha)\ If\ a \equiv b(mod n)\ and\ c \equiv d(mod n), then\ a + c \equiv b + d(mod n).$
$\beta)\ If\ a \equiv b(mod n)\ and\ c \equiv d(mod n), then\ ac \equiv bd(mod n).$

ii. State and prove Fermat's theorem

b) Attempt any **one** of the following: 07

i. Solve the following set of simultaneous congreunces
$$x \equiv 2(mod 3), x \equiv 3(mod 5), x \equiv 2(mod 7).$$

ii. Find the remainder when 15! is divided by 17.

Q3 (a) Attempt any **one** of the following: 05

i. Prove that the functions σ and τ are multiplicative functions.

ii. If $n \geq 1$ and $gcd(a, n) = 1$, then prove that $a^{\phi(n)} = 1(mod n)$.

(b) Attempt any **one** of the following: 05

i. Find $\phi(36000)$.

ii. Prove that any prime of the form $3n + 1$ is also of the form $6m + 1$.

Q4 Choose the correct alternative and **rewrite the sentence:** 10

a) Two integers a and b not both of which are zero are said to be relatively prime, if ----------

i) $gcd\,(a, b) = 0$
ii) $gcd(a, b) = 1$
iii) $gcd(a, b) = a$
iv) $gcd(a, b) = b$

b) If $d = gcd(a, n)$, then the linear congreunce $ax \equiv b(mod n)$ has a solution if and only if ....

i) $d|a$
ii) $d|b$
iii) $b|d$
iv) $a|d$

c) The value of $(30) =$ ......

i) 1
ii) 0
iii) 3
iv) -1

d) If n is even integer, then $\phi(2n) =$

i) $2\phi(n)$
ii) $2n$
iii) n
iv) $\phi(n)$

e) A composite integer n is called a pseudoprime, if ....

i) $n|2^n + 2$
ii) $n\,|2^n - 2$
iii) $n|2^n - 1$
iv) $n\,|2^n$

Total No. of Printed Pages: 3

SUBJECT CODE NO: - Y-2051
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-III)
Examination March / April - 2023
Mathematics MAT - 302 Integral Transforms

[Time:1.30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 a) Attempt any one of the following: 08

i. $If\ L^{-1}\{f(s)\} = F(t)\ and\ L^{-1}\{g(s)\} = G(t)$, then prove that

$$L^{-1}\{f(s)g(s)\} = \int_0^t F(u)G(t-u)du.$$

ii. $If\ L\{F(t)\} = f(s), then\ for\ n = 1,2,3\ \ldots.., prove\ that$

$$L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s).$$

b) Attempt any one of the following: 07

i. If $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(e^{-x}x^n)$, then prove that $L\{L_n(t)\} = \frac{(s-1)^n}{s^{n+1}}, s > 1$.

ii. Using Heavi-side's expansion formula, find $L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}$.

Q2 a) Attempt any one of the following: 08

i. If $F(x)$ has the Fourier transform f (s) , the prove that $F(x)\ cos\ ax$ has the Fourier transform $\frac{1}{2}f(s-a) + \frac{1}{2}f(s+a)$.

ii. If $L\{F(t)\} = f(s)$, then prove that $\lim_{t\to\infty} F(t) = \lim_{s\to 0} s\, f(s)$.

b) Attempt any one of the following: 07

i. Find the Fourier transform of

$$f(x) = \begin{cases} x^2, & if\ |x| < a, \\ 0, & if\ |x| > a \end{cases}$$

ii. Evaluate the integral $\int_0^\infty e^{-ax}x^{m-1}cos\ bxdx$.

Q3 a) Attempt any one of the following: 05

i. If $L\{F(t)\} = f(s)$, then prove that $L\{e^{at}F(t)\} = f(s-a)$.

ii. If $f(s)$ is the Fourier transform of $F(x)$, then prove that $\frac{1}{s} f\left(\frac{s}{a}\right)$ is the Fourier transform of $F(ax)$.

b) Attempt any one of the following: 05

i. Find the value of $L^{-1}\left\{\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right\}$

ii. Prove that $L\{te^{at} \sin at\} = \frac{2a(s-a)}{(s^2-2as+2a^2)^2}$.

Q4 Choose the correct alternative and rewrite the sentence: 10

a) If $0 < l < 1$ $then$ $\Gamma(l)\Gamma(1-l) =$ …………..

i. $\frac{\sin l\pi}{\pi}$

ii. $\frac{\cos l\pi}{\pi}$

iii. $\frac{\pi}{\sin l\,\pi}$

iv. $\frac{\pi}{\cos l\,\pi}$

b) If $L\{F(t)\} = f(s)$ then $L\left\{\frac{F(t)}{t}\right\}$ =…………

i. $\int_0^\infty f(u)du$

ii. $\int_1^\infty f(u)du$

iii. $\int_s^\infty f(u)du$

iv. $\int_{-\infty}^\infty f(u)du$

c) $L^{-1}\left\{\frac{1}{s^2}\right\}$ = …………

i. t^2

ii. t

iii. t^3

iv. 1

d) The finite Fourier sine transform of $f(x) = 1\ for\ 0 < x < \pi$ is ……….

i. $\frac{\pi(-1)^{s+1}}{s}$

ii. $\frac{\pi(-1)^{s-1}}{s}$

iii. $\frac{\pi(-1)^{s+1}}{s^2}$

iv. $\frac{1-(-1)^s}{s}$

e) $L\{cosh\ at\} =$ ……….

i. $\frac{a}{s^2-a^2}$

ii. $\frac{s}{s^2-a^2}$

iii. $\frac{a}{s^2+a^2}$

iv. $\frac{s}{s^2+a^2}$

Total No. of Printed Pages: 03

SUBJECT CODE NO: - Y-2117

FACULTY OF SCIENCE AND TECHNOLOGY

B.Sc. S.Y (Sem-III)

Examination March / April - 2023

Mathematics MAT - 303 Mechanics-I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks. labelled diagrams whenever necessary.

Q1 (a) Attempt any one of the following: 08

i) Find the magnitude and direction of the resultant of any number of the coplanar forces acting at a point.

ii) State and prove the triangle law of forces.

(b) Attempt any one of the following: 07

i. The greatest and least magnitudes of the resultant R of two forces P and Q are G and L respectively. Show that $R^2 = G^2 \, cos2\theta + L^2 \, sin2\theta$, where 2θ is the inclination between the two forces P and Q.

ii. A uniform plane lamina in the form of rhombus, one of whose angle is 120°, is supported by two forces applied at the centre in the direction of the diagonals so that one side of the rhombus is horizontal. Prove that if P and Q be the forces and $P > Q \text{ then } P = \sqrt{3}Q$.

Q2 a) Attempt any one of the following: 08

i. Prove that the magnitude of the moment of the couple equals to the product of magnitude of a force in the couple and arm of the couple.

ii. Prove that the sum of the vector moments of two like parallel force acting on a rigid body about any point equals to the vector moment of their resultant about the same point.

b) Attempt any one of the following: 07

i. Three forces of magnitudes P, Q, R act along the sides BC, CA, AB of a triangle ABC, taken in order; prove that if the resultant passes through the circumcenter of AABC, then

$$P\ cos\ A + Q\ cos\ B + \ R\ cos\ C\ =\ 0$$

iii. Three rods of unequal lengths are joined to form a AABC. If the masses of the sides a, b, c be proportional to $(b + c - a), (c + a - b)\ and\ (a + b - c)$. Prove that the C.G. is incentre.

Q3 (a) Attempt any one of the following: 05

i. If a system of parallel forces of magnitudes F_1, F_2, ... Fn act at some given n points, then prove that the resultant of these forces act through their centre.

ii. A system of coplanar forces acting at a point is in equilibrium if and only if the algebraic sum of the resolved parts of the given forces along any two mutually perpendicular directions must separately vanish.

(b) Attempt any one of the following: 05

i. Find the vector moment of a force $\vec{F} = \vec{\imath} + 2\vec{\jmath} + \ 3\vec{k}$ acting at a point (-1, 2, 3) about origin.

ii. If a transversal cuts the lines of action of the concurrent forces P, Q and R in A, B and C respectively, R being the resultant of the two forces P and Q. Show that

$$\frac{P}{OR} + \frac{Q}{OB} = \frac{R}{OC}$$

Q4 Choose the correct alternative and rewrite the sentence: 10

(a) Two forces are said to be like parallel forces when they act in the .... and their line of action do not meet at a point.

i. any direction

ii. opposite direction

iii. same direction

iv. circular direction

(b) A particle is a body which is indefinitely small in

i. not size and not shape

ii. only shape

iii. only size

iv. size and shape

(c) The centre of the gravity is

i. dependent

ii. independent

iii. not unique

iv. unique

(d) The resolved part of the force R along the direction of the unit vector e is

i. $\vec{R} + \vec{e}$

ii. $\vec{R}\ o\ \vec{e}$

iii. $\vec{R} - \vec{e}$

iv. $\vec{R}\ x\ \vec{e}$

(e) If two forces of magnitude P each acting at an angle then the magnitude R of their resultant force is given by ....

i. $R = 2P \cot\frac{\theta}{2}$

ii. $R = 2P \tan\frac{\theta}{2}$

iii. $R = 2 \sin\frac{\theta}{2}$

iv. $R = 2P \cos\frac{\theta}{2}$

Total No. of Printed Pages: 2

SUBJECT CODE NO: - Y-2065
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y Sem-IV
Examination March / April - 2023
Mathematics MAT - 401 Numerical Methods

[Time: [1.30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) Attempt all questions
2) Figure to the right indicate full marks
3) Use of non-Programmable calculator and logarithmic table is allowed

Q1 A) Attempt any one 08

a) Explain the bisection method for finding real roots of an equation f(x)=0
b) Derive Newton's forward difference interpolation formula

B) Attempt any one 07

c) Use the Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$ which lies between 2 and 3

d) Using Newton divided difference formula find f(x) as a polynomial in x. given data is

x	-1	0	3	6	7
f(x)	3	-6	39	822	1611

Q2 A) Attempt any one 08

a) Explain the method of fitting a straight line $Y = a_o + a_1 x$
b) Explain the method of factorization to solve the system of linear equations

B) Attempt any one

c) Determine the constant a and b by the method of least squares 07
Such that $y = ae^{bx}$ fits the following data

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

d) Solve the following system
2x+y+z=10
3x+2y+3z=18
x+4y+9z=16
By Gaussian elimination method

Q3 A) Attempt any one 05

a) Explain Taylor's series method to solve the differential equation

$$y' = f(x, y)$$
$$with\ the\ initial\ condition\ y(xo) = y_o$$

b) With the usual notations prove that

$$\mu \equiv \left[1 + \frac{1}{4}\delta^2\right]^{\frac{1}{2}}$$

B) Attempt any one

c) Using Picard's method obtain the solution of 05

$$\frac{dy}{dx} - 1 = xy\ with\ y(0) = 1$$

And compute y(0.1) correct to four decimal places

d) Show that

$$e^x\left(u_o + x\Delta u_0 + \frac{x^2}{2!}\,\Delta^2 u_o + ----\right)$$
$$= u_0 + u_1 x + u_2\frac{x^2}{2!} + ----$$

Q4 Choose the correct alternative 10

i) Which of the following is transcendental equation?

a) $x^3 - x - 1 = 0$ b) $x^3 + x + 1 = 0$
c) $x^3 - 2x^2 + 1 = 0$ d) $xe^x + sinx = 0$

ii) If δ is central difference operator then $\delta y_{\frac{3}{2}}$ = ------

a) $y_1 - y_o$ b) $y_2 - y_1$ c) $y_3 - y_2$ d) $y_4 - y_3$

iii) If $y(x) = 2x^2 + x - 1$, then $\Delta^3 y(x)$ is ----

a) 0 b) 1 c) 2 d) 3

iv) The chebyshev polynomial of degree two is ------

a) 1 b) x c) $2x^2 - 1$ d) $2x^2 + 1$

v) If $A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$ then the characteristic polynomial is -------

a) $\lambda^2 + \lambda - 2$ b) $\lambda^2 - \lambda + 2$ c) $\lambda^2 + 2\lambda + 1$ d) $\lambda^2 - 2\lambda - 1$

Total No. of Printed Pages: 3

SUBJECT CODE NO: - Y-2066
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-IV)
Examination March / April - 2023
Mathematics MAT - 402 Partial Differential Equation

[Time: 1.30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory
2) Figures to the right indicate full marks

Q1 A) Attempt any one

a) Explain the method of solution of the partial differential equation 08

$$f(z,p,q) = 0 \text{ and solve } p^3 + q^3 = 27z$$

b) Define Lagrange's Linear partial differential equation. Obtain subsidiary equation of Lagrange's partial differential equation 08

B) Attempt any one

c) Solve : 07

$$(y^2 + z^2 - x^2)p - 2xyq + 2zx = 0$$

d) Find the complete integral of 07

$$(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$$

Q2 A) Attempt any one

a) Explain the charpit's method for solution of partial differential equation $f(x,y,z,p,q) = 0$ 08

b) Explain the method of solution of $Rr + Ss + Tt + f(x,y,z,p,q) = o$ when $S^2 - 4RT > 0$ where R,S,T are the continuous functions of x,y and possesses continuous partial derivatives of any order 08

B) Attempt any one

c) Find the complete integral of $x_3^2\, p_1^2\, p_2^2\, p_3^2 + p_1^2\, p_2^2 - p_3^2 = 0$ by Jacobi's method 07

d) Solve $(D^2 - DD' + D' - 1)z = \cos(x+2y) + e^y$ 07

Q3 A) Attempt any one 05

a) With usual notations prove that $\frac{1}{F(D,D')}(e^{ax+by}V) = e^{ax+by}\frac{1}{F(D+a,D'+b)}(v)$

b) Find the general solution of $(D - m_o D')^2 z = 0$

B) Attempt any one 05

c) Solve $\frac{\partial q}{\partial y} - \frac{1}{y}z = x$

d) Find the general solution of the equation $(D + D')z = sinx$

Q4 Choose the correct alternative 10

1) Auxiliary equation of

$$(y^2 + z^2 - x^2)p - 2xyq + 2xz = O\ are - - - - -$$

a) $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$

b) $\frac{dx}{y^2+z^2-x^2} = \frac{dy}{-2xy} = \frac{dz}{-2xz}$

c) $\frac{dx}{y^2+z^2+x^2} = \frac{dy}{-2xy} = \frac{dz}{2xz}$

d) $\frac{dx}{y^2+z^2-x^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$

2) The symbols r,s and t are denoted respectively by -------

a) $\frac{\partial z}{\partial x}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}$

b) $\frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial x^2}$

c) $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2}$

d) None of these

3) The complete integral of $p^2 + q^2 = n^2 is -----$

a) $z = ax + \sqrt{(n^2 - a^2)}\,.\,y + c$

b) $z = ax + \sqrt{(n^2 - a^2)y} + c$

c) $z = ax + \sqrt{n^2 - a^2} + c$

d) $z = \sqrt{(n^2 - a^2)}\ y + c$

4) Complementary function of

$$(D^2 - 2aDD' + a^2\,D'^2)z = f(y + ax)\ is ----$$

a) $z = \phi_1(y + ax) + \phi_2(y + ax)$

b) $z = \phi_1(y + ax) + x\phi_2(y + ax)$

c) $z = \phi_1(y + ax) - x\ \phi_2(y + ax)$

d) $z = \phi_1(y - ax) + \mathrm{x}\ \phi_2(y - ax)$

5) Solution of $S = 2x + 2y$ is ------

a) $z = x^2y + xy^2 + F(y) + f(x)$

b) $z = xy + xy^2 + F(y) + f(x)$

c) $z = x^2y - xy^2 + F(y) + f(x)$

d) $z = x^2y + xy^2 - F(y) - f(x)$

Total No. of Printed Pages: 2

SUBJECT CODE NO: - Y-2125
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. S.Y (Sem-IV)
Examination March / April - 2023
Mathematics MAT - 403 Mechanics-II

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) Attempt all questions.
2) Figures to the right indicate full marks.
3) Draw well labeled diagram whenever necessary.

Q1 A) Attempt any one: 08

a) Find the radial and transverse components of velocity.
b) Find the expressions for velocity and acceleration in terms of vector derivatives.

B) Attempt any one: 07

a) A point moves in a curve so that its tangential and normal accelerations are equal and the tangent rotates with uniform angular velocity. Show that the intrinsic equation of Path is of the forms

$$S = \mathrm{A}.e^{\psi} + \mathrm{B}$$

b) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. show that the velocity of recoil is

$$\left[\frac{2\mathrm{m}^2\,\mathrm{gh}}{\mathrm{M}(\mathrm{m}+\mathrm{M})}\right]^{1/2}$$

Q2 A) Attempt any one 08

a) prove that the kinetic energy of particle of mass m moving with velocity is $\vec{v}$ is $\frac{1}{2}$ mV^2 Also prove that change in kinetic energy of the particle is equal to the work done.
b) Find the differential equation of the Path of a particle moving under a central farce f(r). directed towards a fixed point 0.

B) Attempt any one: 07

a) A particle is thrown over a triangle from one end of the horizontal base and grazing over the vertex. It falls on the other end of the base. If A, B be the base angles of the triangle and α the angle of projection.
Prove that: tan α = tan A + tan B
b) A particle of mass 0.1 lb has the velocity $2\vec{\imath} + 3\vec{\jmath}$ ft/sec. at t = 2 sec. It is Subjected to a force $3t^2\vec{\imath} + cos\,(\pi t)\vec{\jmath}$. Find the impulse of the force over the internal $2 \leq t \leq 3$. Also find the velocity at t = 3sec.

Q3 A) Attempt any one: 05

a) Prove that in central orbits the areal velocity is uniform.

b) Find the vertex and the latus rectum of the parabola.

B) Attempt any one: 05

a) A man can throw a cricket ball up to 160 metres and no more. With what speed, in metre per Sec., must it be thrown? (Take g = 980 cm/sec^2)

b) Find the work done by the force $\vec{F} = 2x\vec{\imath} + 2y\vec{\imath}$ in moving a particle from P (1,2) to Q (3,2)

Q4 Choose the correct alternative and rewrite the sentence: 10

i) If the force is acting towards a fixed Point then it is called______

a) central repulsive force. b) Tangential force.
c) Terminal force. d) central attractive force.

ii) The effect of couple acting on the body producess______

a) only a motion of rotation.
b) only a motion of translation.
c) motion of rotation as well as translation
d) None of these.

iii) The time rate of change velocity is called as_____

a) A Speed b) Acceleration c) Displacement. d) Areal velocity.

iv) In central orbits the areal velocity is ______

a) unit b) zero c) Variable d) uniform.

v) The magnitude of velocity is called_____

a) Acceleration. b) Displacement. c) Speed d) Vector

Total No. of Printed Pages: 03

SUBJECT CODE NO: - Y-2047
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V)
Examination March / April - 2023
Mathematics MAT - 502
Abstract Algebra - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one: 08

a) If H is a subgroup of a group G, then for a, $b \in G$ prove that the relation $a \equiv b$ mod H is an equivalence relation.

b) If ϕ is a homomorphism of a group G into group $\bar{G}$ with Kernel K, then prove that K is a normal subgroup of G.

B) Attempt any one: 07

c) If G is a group in which
$(a.b)^i = a^i.b^i$
for three consecutive integers i, for all a, $b \in G$, show that G is abelian.

d) Let G be a group and g is a fixed element in G. Define $\phi: G \to G$ by $\phi(x) = gx9^{-1}$. Prove that ϕ is an isomorphism of G onto G.

Q2 A) Attempt any one: 08

a) If ϕ is a homomorphism of a ring R into ring $R'$with Kernel $I(\phi)$, them prove that
   i. $I(\phi)$ is a subgroup of R under addition.
   ii. If $a \in I(\phi)$ and $r \in R$, then both ar and ra are in $I(\phi)$

b) Prove that if R is a commutative ring with unit element whose only ideals are (0) and R itself, then R is a field.

B) Attempt any one: 07

c) Prove that any field is an integral domain.

d) If U and V are ideal of R, and if
$U + V = \{u + v \,/\, u \in U \text{ and } v \in V\}$
Prove that U+V is also an ideal.

Q3 A) Attempt any one: 05

a) If H and K are subgroups of a group G and $O(H) > \sqrt{O(G)}, O(K) > \sqrt{O(G)}$, then prove that $H \cap K \neq (e)$.

b) If R is a commutative ring with unit element 1 and R/U is quotient ring then prove that
   i. R/U is commutative
   ii. R/U has a unit element 1+U

B) Attempt any one: 05

c) If N and M are normal subgroups of a group G, prove that NM is also a normal subgroup of G.

d) If R is ring with unit element 1 and ϕ is a homomorphism of R into R' prove that $\phi(1)$ is the unit element of R'

Q4 Choose the correct alternative: 10

i. If N is normal subgroup of a group G such that 0(G)=6 and 0(N)=3, then $0(G/N)$ = _______
   a) 3
   b) 2
   c) 9
   d) 18

ii. For any two elements a and b of a group G, if $(a.b)^2 = a^2.b^2$, then G is ______
   a) Abelian group
   b) Quaternion group
   c) Quotient group
   d) None of these

iii. If G is a group and for $x \in G, 0(x) = n$ and $x^m = e,$ then ______

a) m=0
b) m divides n
c) n divides m
d) none of these

iv. If an integral domain D is of finite characteristic, then its characteristic is _____

a) A composite number
b) A prime number
c) Any integer
d) None of these

v. If M is a maximal ideal of a commutative ring R with unit element, then ____

a) R/M is a field
b) R/M is not a field
c) R is a field
d) None of these

Total No. of Printed Pages: 02

SUBJECT CODE NO: - Y-2114
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V)
Examination March / April - 2023
Mathematics MAT-503 Mathematical Statistics - I

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one:

a) State and prove the formula for combined mean of series. 08

b) Explain "Ogive" with suitable example. 08

B) Attempt any one:

c) Find the geometric mean of the following frequency distribution: 07

Marks	0-10	10-20	20-30	30-40
No. of Students	5	8	3	4

d) Find the harmonic mean of the following frequency distribution: 07

Class Interval	2-4	4-6	6-8	8-10
Frequency	20	40	30	10

Q2 A) Attempt any one:

a) Establish the relationship between mean square deviation and variance. 08

b) Explain the various measures of dispersion with their merits and demerits. 08

B) Attempt any one.

c) For 100 observations, the mean and variance were obtained as 19 and 9 respectively. Latter on it was discovers that the observation 12 was misread as 21. Calculate the correct mean and variance of the actual data. 07

d) The first three moments of a distribution about the value 5 of the variables are 2, 20 and 40. Find the mean , variance and third moment about mean. 07

Q3 A) Attempt any one:

a) Define: 05

i. Median
ii. Mode
iii. Moments
iv. Probability
v. discrete random variable

b) prove that the algebraic sum of the deviations of all the variate values from their arithmetic mean is zero. 05

B) Attempt any one:

c) a problem in mathematics is given to three students A,B,C whose chances of solving it are $\frac{1}{2}$,$\frac{1}{3}$ and $\frac{1}{4}$ respectively. 05

What is the probability that the problem will be solved?

d) If a random variable X has the probability density function as follows: 05

$$f(x) = \frac{1}{4} \quad ,-2 < x < 2 ,$$
$$= 0 , otherwise$$

$Find : i) \ P \ (x < 1)$

$ii) \ P \ [(2x + 3) > 5]$

Q4 Choose the correct alternative of the following: 10

1) The skewness when mean = 4, median = 4 , and standard deviation = 3 is ______

a) 1 b) -1 c) 0 d) 2

2) If P(A) = 0.37 , P(B) = 0.48 and $P(A \cup B) = 0.85$ then the value of $P(A \cap B) is$ _____

a) 3.7 b) 4.8 c) 8.5 d) 0

3) The value of the variable correspond to maximum frequency is known as ______

a) Median b) Mode c) Mean d) Harmonic mean

4) The geometric mean of 6 , 24 is ______

a) 12 b) 13 c) 14 d) 11

5) The sum of the squares of the deviations of all the values taken about their arithmetic mean is ______

a) Zero b) Maximum c) Minimum d) Infinite

Total No. of Printed Pages: 3

SUBJECT CODE NO: - Y-2115
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-V)
Examination March / April - 2023
Ordinary Differential Equation -I 504

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figure to the right indicate full marks.

Q1 A) Attempt any one:

a) Suppose a and b are continuous functions an interval I. Let A be a function such that A'= a. 8
The function Ψ given by
$\Psi(x) = e^{-A(x)} \int_{xo}^{x} e^{A(t)}\, b(t)dt.$
Where x_o is in I, is a solution of the equation $y' + a(x)y = b(x)$ on I.
The function ϕ, given by $\phi, (x) = e^{-A(x)}$ is a solution of the homogeneous equation $y' + a(x)y = 0$.
Prove that if c is any constant,
$\phi = \Psi + c\,\phi$, is a solution of $y' + a(x)y = b(x)$.

b) Consider the equation $y' + ay = 0$, where a is a complex constant if c is any complex number. Prove that the function ϕ define by 8
$\phi(x) = c\, e^{-ax}$ is a solution of the equation $y' + ay = 0$.

B) Attempt any one:

c) Find all solution of the equation 7
$y' - 2y = x^2 + x.$

d) Consider the equation $Ly' + Ry = Ee^{iwx}$, where L,R,E,W are positive constant. 7
Find the solution ϕ which satisfies $\phi(0) = 0$.

Q2 A) Attempt any one:

a) If ϕ_1 , ϕ_2 are two solution of $y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_o then prove th 8

$$w(\phi_1, \phi_2)(x) = e^{-a1(x-x_o)}.\, w(\phi_1, \phi_2)(x_0)$$

b) Let a_1, a_2 be constants and consider the equation $L(y) = y" + a_1y' + a_2y = 0$. 8
If r_1, r_2 are distinct roots of the characteristics polynomial P, where $P(r) = r^2 + a_1r + a_2$ then prove that the function ϕ_1, ϕ_2 define by $\phi_1(x) = e^{r_1x}, \phi_2(x) = e^{r_2x}$ are solution of $L(y) = 0$.

B) Attempt any one:

c) Find all solutions of the equation
$y" - 2y' - 3y = 0, \quad y(0) = 0, \quad y'(0) = 1.$ 7

d) Determine whether the functions $\phi_1(x) = cosx, \quad \phi_2(x) = 3(e^{ix} + e^{-ix})$ are linearly dependent or independent. 7

Q3 A) Attempt any one:

a) If z_1 *and* z_2 are two complex numbers then prove that 5
$||z_1| - |z_2|| \le |z_1 - z_2|$

b) Prove that $e^{i\theta} = cos\theta + isin\theta$. 5

B) Attempt any one:

c) If r is such that $r^3 = 1$ and $r \neq 1$. Prove that $1 + r + r^2 = 0$. 5

d) If $z = x + iy$, where x, y are real, show that $|e^z| = e^x$. 5

Q4 Choose the correct alternative of the following. 10

1) $\frac{1+i}{1-1} =$ ___
a) -1 b) 0 c) i d) None of these

2) All solution of the equation $y" = x^2. on -\infty < x < \infty$
a) $\phi(x) = \frac{x^4}{12} + Gx^2 + C_2$.
b) $\phi(x) = \frac{x^4}{12} + Gx + C_2$.
c) $\phi(x) = \frac{x^3}{12} + Gx + C_2$.
d) None of these.

3) If $\phi_1(x) = cosx, \phi_2(x) = sinx$ then $w(\phi_1, \phi_2)(x) =$……..
a) 0 b) 1 c) cosx+sinx d) None of these

4) All solutions of the equation $y'' - 4y = 0$.
   a) $\phi(x) = Ge^{4ix} + c_2e^{-4ix}$.
   b) $\phi(x) = Ge^{4x} + c_2e^{-4x}$
   c) $\phi(x) = Ge^{-4ix} + c_2e^{-4x}$
   d) None of these

5) $\phi(x) = sin2x\ is\ a\ solution\ of\ the\ equation.$
   a) $y'' + 4y = 0$
   b) $y'' -\ 4y = 0$
   c) $y'' + 2y = 0$
   d) None of these

Total No. of Printed Pages: 02

SUBJECT CODE NO: - Y-2061
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Mathematics MAT-601 Real Analysis-II

[Time: 1: 30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Prove any one: 08

a. Prove that every open subset G of R' can be written $G = \cup I_n$, where I_1, I_2 .... Are a finite number or a countable number of open intervals which are mutually disjoint..

b. Let $< M_1, P_1 > and < M_2, P_2 >$ be metric spaces, and let $f: M_1 \to M_2$ Then prove that f is continuous on M_1 if and only if $f^{-1}(F)$ is closed subset of M_1 whenever F is a closed subset of M_2

B) Attempt any one 07

c. For $P < x_1, y_1 >$ and $< x_2, y_2 >$, define $\sigma(P, Q) = |x_1 - x_2| + +y_1 - y_2|$, show that σ is a metric for the set of ordered pairs of real numbers.

d. Let f be the function from $R^2 onto\ R^1$ defined by
$f(< x, y >) = x \quad (< x, y > \in R^2)$
show that f is contnuous on R^2

Q2 A) Attempt any one 08

a. Let $< M_1, P_1 >$ be a compact metric space if f is a continuous function from M_1 into a metric space $< M_2, P_2 >$, then prove that f is uniformly continuous on M_1

b. If f is continuous on the closed bounded interval [a, b], and if

$$F(x) = \int_a^x f(t)dt \quad (a \le x \le b),$$

Then prove that $F'(x) = f(x) \quad (a \le x \le b)$

B) Attempt any one 07

c. Prove that every finite subset of any metric space is compact.

d. Find the Fourier series for the function $f(x) = e^x \ in \ -\pi < x < \pi$

Q3 A) Attempt any one 05

a. if A is a closed subset of the compact metric space < M, P>, then prove that the metric space <A, P> is also compact.

b. If $f \in R[a, b], \quad g \in R[a, b], \ and\ if\ f(x) \le g(x)$ almost every where $(a \le x \le b)$ then prove that $\int_a^b f \le \int_a^b g$

B) Attempt any one 05

c. Let $f(x) = x\ (0 \le x \le 1), Let\ \sigma\ be$ the subdivision $\{0, \frac{1}{3}, \frac{2}{3}, 1\}$ of [0,1] compute $L\ [f; \sigma]$

d. If $0 \le x \le 1$ show that

$$\frac{x^2}{\sqrt{2}} \le \frac{x^2}{\sqrt{1+x}} \le x^2$$

Q4 Choose the correct alternative 10

1. The function P defined by $p(x, y) = |x - y|$ is a metric for the set R of real numbers, then the metric space $< R, P >$ is denoted by ____
   a. Rd b. R^d c. R^1 d. R^∞

2. Every singleton set in a discrete metric space Rd is _______
   a. Open b. closed c. open and closed d. none of these

3. The metric space R^1 is -------
   a. Not complete
   b. Totally bounded
   c. Complete but not totally bounded
   d. Complete and totally bounded

4. If f is Riemann integrable function on [a,b] and a<c<b, then _______
   a. $\int_a^b f > \int_a^c f + \int_c^b f$
   b. $\int_a^b f < \int_a^c f + \int_c^b f$
   c. $\int_a^b f = \int_a^c f - \int_c^b f$
   d. $\int_a^b f = \int_a^c f + \int_c^b f$

5. When m=n, for n=0,1, 2 ….
   $$\int_{-\pi}^{\pi} \cos mx \cos nx \, dx = \cdots$$
   a. 0 b. 1 c. $-\pi$ d. π

Total No. of Printed Pages: 02

SUBJECT CODE NO: - Y-2062
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Mathematics MAT - 602 Abstract Algebra - II

[Time: 1 :30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figure to the right indicate full marks.

Q1 A. Attempt any one: 08

a. If T is homomorphism of a vector space U onto a vector space V with kernel W, then prove that V is isomorphic to U/W

b. Prove that if $v_1, v_2, \ldots v_n$ are in vector space V. then either they are linearly independent or some V_k is a linear combination of the preceding ones, $v_1, v_2, \ldots v_{k-1}$

B. Attempt any one 07

c. Let F be the field of all real numbers and let V be set of all sequences

$$\{(a_!, a_2, \ldots a_n, \ldots), |a_i^2 \varepsilon F\}$$

$If\ U = \{(a_!, a_2, \ldots a_n, \ldots), EV \mid \sum_{i=1}^{\infty} a_i^2\ \ is\ finite\}$ then prove that U is a subspace of V.

d. If T is an isomorphism of vector space V onto vector space W, then prove that T maps a basis of V onto a basis of W.

Q2 A. Attempt any one: 08

a. If W is subspace of finite-dimensional vector space V over F, then prove that $A\big(A(W)\big) = W$.

b. Prove that if V is finite-dimensional inner product space, then V has an orthonormal set as a basis.

B. Attempt any one: 07

c. Let V be the set of all continuous complex-valued function on the closed unit interval [0,1]. If $f(t), g(t) \in V$, such that

$$(f(t), g(t) = \int_0^1 f(t)$$

Prove that this define an inner product on V.

d. If A and B are submodules of on R Modules M, then prove that $A + B = \{a + b \mid a \in A, b\ \in A\}$ is a submodule of M.

Q3 A. Attempt any one: 05

a. If W is a subspace of an inner product space V, then prove that $W^{\perp}$ is a subspace of V.

b. If V is vector space over F and $v_1, v_2, \ldots, v_n \in V$ are linearly independent then prove that every element in their linear span has a unique representation in the from $\lambda_1 V_1 + \lambda_2 V_2 +$ ___$V_n V_n$ with the $\lambda_i EF$.

B. Attempt any one: 05

c. If V is finite-dimensional and $W_1$and $W_2$are subspaces of V, describe $A(W_1 \cap W_2)$ in terms of $A(W_1)$and $A(W_2)$

d. If F is the field of real numbers, prove that the vectors (1, 1, 0, 0), (0,1, -1, 0) and (0, 0,0, 3) in $F^{(4)}$ are linearly independent over F.

Q4 Choose correct alternatives: - 10

1. In an inner product space V, the inequality $|(u, v)| \leq ||u||$.‖v‖ is called ____
   a. Triangle inequality
   b. Bessel's inequality
   c. Schwarz inequality
   d. none of these

2. If V is a finite decisional vector space and $\hat{v}$ is its dual space then _____
   a. Dim $\hat{v} = sim\ V$ b. Dim $\hat{v} > sim\ V$ c. Dim $\hat{v} < sim\ V$ d. none of these

3. A subset S of a vector space V over F form basis if S is linearly independent and ___
   a. L(S)=S b. L(S)=V c. L(S)=F d) none of these

4. Every subspace of a vector space V other than (0) and V is called _______
   a. Improper subspace b. proper subspace c. dual space d. none of these

5. Vector space is defined over a _______
   a. Monoids b. group c. ring d. field

Total No. of Printed Pages: 02

SUBJECT CODE NO: - Y-2122
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Mathematics
Mathematical Statistics-II - MAT -603

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one: 08

a) Prove that:
The mathematical expectation of the product of a number of independent random variables is equal to the product of their expectations.

b) If $x_1, x_2 \dots \dots \dots \dots . x_n$ be n random variables, then show that

$$V\left(\sum_{i=1}^{n} a_i \; x_i\right) = \sum_{i=1}^{n} a_i^2 \, V\,(x_i) + 2\sum_{i=1}^{n}\;\sum_{j=1}^{n} a_i \, a_j \; cov(x_i, x_j)$$

B) Attempt any one: 07

c) If m things are distributed among 'a' men and 'b' women, show that the probability that the number of things received by men is odd is given by

$$\frac{1}{2}\left[\frac{(b+a)^m - (b-a)^m}{(b+a)^m}\right]$$

d) If x be a random variable with the following probability distribution:

X:	-3	6	9
P(x=x):	$^1/_6$	$^1/_2$	$^1/_3$

Find E(x) and E (x^2) and using the laws of expectation evaluate $E(2x+1)^2$

Q2 A) Attempt any one: 08

a) Find the mode of the normal distribution.

b) In case of uniform distribution , Prove that : $\mu_2 = \frac{1}{12}(b-a)^2$

B) Attempt any one: 07

c) If $x \sim B\,(n, p)$, show that:

$$E\left(\frac{x}{n}-p\right)^2=\frac{pq}{n}\ ;Cov\ \left(\frac{x}{n},\frac{n-x}{n}\right)=\frac{-pq}{n}$$

d) If x and y are independent Poisson variates with means $\lambda_1 and\ \lambda_2$ respectively
Find i) x + y = k ii) x = y

Q3 A) Attempt any one — 05

a) Prove that correlation coefficient is the geometric mean between the regression coefficients.

b) Find the mean and variance of the Poisson distribution

B) Attempt any one: — 05

c) If x and y are independent poisson variates having mean 1 and 3 respectively. Find the variance of 3x + y.

d) If the independent random variables x ,y are binomially distributed , respectively n = 3 , $P=\frac{1}{3}$ and n = 5, $P=\frac{1}{3}$, write down the probability that $x+y\ \geq 1$

Q4 Choose the correct alternatives: — 10

1) If x is a random variable , also a and b are constants , then $V(ax+b)=\cdots$
   a) $a^2\,V(x)$ b) $av\ (x)+b$ c) $V\ (a^2x)+b$ d) None of these

2) If x and y are independent the cov (x,y) = ………..
   a) 1 b) 0 c) -1 d) 2

3) When the correlation coefficient $r=\pm 1$ then the two regression lines ……
   a) Are perpendicular to each other
   b) Coincide
   c) Are parallel to each other
   d) Do not exist

4) If $x\sim p\ (\lambda)$ then mean of poisson distribution is …………
   a) λ^2 b) $1/_{\lambda}$ c) λ d) $1/_{\sqrt{\lambda}}$

5) The mean of the binomial distribution is ………..
   a) Np b) npq c) $npq\ (q-p)$ d) $npq\ \{1+3(n-2)\ pq\}$

Total No. of Printed Pages: 03

SUBJECT CODE NO: - Y-2123
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Ordinary Differential Equation-II - MAT- 604

[Time: 1:30 Hours] **[Max. Marks: 50]**

Please check whether you have got the right question paper.

N. B 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q1 A) Attempt any one: 08

a) Let $\phi_1, \phi_2, \ldots\ldots, \phi_n$ be the n solutions of
$L(y) = y^n + a_1\, y^{(n-1)} + --- + a_n(x)y = 0$ on I satisfying
$\phi_i^{(i-1)}(x_0) = 1 \;\; , \phi_i^{(j-1)}(x_0) = 0 \;,\; j \neq i$
Prove that ϕ is any solution of L(y)=0 on I, three are n constant $C_1, C_2 --- C_n$
Such that $\phi = C_1\,\phi_1 + C_2\,\phi_2 + \; -- + C_n\phi_n$

b) Let $\phi_1, \phi_2 \ldots \;, \phi_n$ be n solutions of
$L(y) = y^{(n)} + a_1(x)y^{(n-1)} + --- + a_n(x)y = 0$ on Interval I , and let x_0 be any point in I then Prove that $W(\phi_1, \phi_2, \ldots., \phi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1(+)dt\right]$
$W(\phi_1, \phi_2, \ldots.\phi_n)(x_0)$

B) Attempt any one 07

c) Consider the equation

$$y'' + \frac{1}{x}\; y' + \frac{1}{x^2}\; y = 0\; for\; x > 0$$

I. Show that there is a solution of the form x^r, where r is constant.

II. Find two linearly independent solutions for $x > 0$ and prove that they are linearly independent.

III. Find two solutions ϕ_1, ϕ_2 satisfying
$\phi_1(1) = 1 \quad , \quad \phi_2(1) = 0$
$\phi_1^1(1) = 0 \quad , \quad \phi_2^1(1) = 1$

d) Find all solutions of
$xy'' - (x+1)y' + y = 0$ given that one solutions is $\phi_1(x) = e^x (x > 0)$

Q2 A) Attempt any one 08

a) Let b be continuous on an interval I. Let $\phi_1, \phi_2, \ldots, \phi_n$ be the basis for the solution of $L(y) = y^{(n)} + a_1(x)\, y^{(n-1)} + \ldots.. + a_n(x) y = 0$
Prove that every solution ψ of $L(y) = b(x)$ can be written as:
$\psi = \psi_p + C_1\phi_1 + C_2\phi_2 + \cdots + C_n\phi_n$
Where ψ_p is particular solution of $L(y) = b(x)$ and $C_1, C_2 \ldots C_n$ are constants.
Every such ψ is solution of $L(y) = b(x)$
A particular solution $\psi(P)$ is given by

$$\psi_p = \sum_{k=1}^{n} \phi_k(x) \int_{x_0}^{x} \frac{W_k(+)b(+)}{W(\phi_1, \phi_2 \ldots \phi_n)t}\, dt$$

b) Consider the second order Euler equation $x^2y'' + axy' + by = 0$ (a ,b constant) and polynomial q is given by $q(r) = r(r-1) + ar + b$
Prove that basis for the solution of Euler equation on any interval not containing x = 0 is given by $\phi_1(x) = |x|^{r1}, \phi_2(x) = |x|^{r_2}$ in case r_1& r_2 are distinct root of q.

B) Attempt any one: 07

c) Show that there is basis ϕ_1, ϕ_2 for the solution of
$xy'' + 4xy' + (2 + x^2)y = 0\ (x > 0)$
of the form $\phi_1(x) = \frac{\psi_1(x)}{x^2}, \phi_2(x) = \frac{\psi_2(x)}{x^2}$

d) Find the linearly independent power series solution of the equation $y'' - xy = 0$

Q3 A) Attempt any one : 05

a) Show that

$$\int_{-1}^{1} P_n(x)\, P_m(x) = 0 \quad (n \neq m)$$

b) Find all solutions of the equation $x^2y'' + 2xy^1 - 6y = 0 \quad (x > 0)$

B) Attempt any one 05

c) Find the singular point of the equation $x^2y'' + (x + x^2)\, y^1 - y = 0$ and determine those which are regular singular point.

d) Find all solutions ϕ of the form
$\phi(x) = |x|^r \sum_{k=0}^{\infty} C_k\, x^k\ (|x| > 0)$ for the equation
$x^2y'' + xy' + \left(x^2 - {}^1/_4\right) y = 0$

Q4 Choose the correct alternative 10

I. If $\phi_1 , \phi_2 \ldots \ldots , \phi_n$ are n solutions of
$L(y) = y^{(n)} + a_1(x)\, y^{(n-1)} + \cdots a_n(x)y = 0$ an interval I, then they are linearly independent if and only if ……..

a) $W\,(\phi_1 , \phi_2 \ldots\ldots \phi_n)(x) = 0 \;\; \forall x \in I$
b) $W(\phi_1 , \phi_2 \ldots \ldots \; \phi_n)(x) \neq 0 \;\; \forall x \in I$
c) $W(\phi_1 , \phi_2 \ldots\ldots\ldots \phi_n)(x) = \exp\left[- \int_{x_0}^{x} a_1(+)dt\right]$
d) $W\,(\phi_1 , \phi_2 \ldots \ldots \phi_n)(x) = \exp\left[\int_{x_0}^{x} a_1(+)dt\right]$

II. One solution of the equation $y'' - \frac{2}{x^2}\, y = 0 \; (0 < x < \infty)$ is ………

a) $\phi(x) = x^2$
b) $\phi(x) = x$
c) $\phi(x) = e^x$
d) $\phi(x) = e^{-x}$

III. The singular point of the equation
$a_0(x)y^n + a_1(x)y^{(n-1)} + \cdots + a_n(x)y \; is\; the\; point\; x_0 for\; which \ldots\ldots$

a) $a_0(x_0) \neq 0$
b) $a_1(x_0) = 0$
c) $a_0(x_0) = 0$
d) $a_1(x_0) \neq 0$

IV. $\int_{-1}^{1} P_n^2(x)dx = \ldots\ldots\ldots$

a) ${}^{3}\!/_{2n+1}$
b) ${}^{1}\!/_{2n+1}$
c) ${}^{2}\!/_{2n+1}$
d) ${}^{2}\!/_{2n-1}$

V. The equation $x^2y'' + xy' + (x^2 - a^2)y = 0$ is

a) Euler equation
b) Legendre equation
c) Nonhomogeneous equation
d) Bessel equation

Total No. of Printed Pages: 02

SUBJECT CODE NO: - Y-2124
FACULTY OF SCIENCE AND TECHNOLOGY
B.Sc. T.Y (Sem-VI)
Examination March / April - 2023
Programming in C-II- MAT-605

[Time: 1:30 Hours] **[Max. Marks: 40]**

Please check whether you have got the right question paper.

N. B
1) All questions are compulsory.
2) Assume the data wherever not given with justification.
3) Figures to the right indicate full marks.

Q1 A) Attempt any one: 05

a) State dangling else problem? How to resolve it?

b) Discuss rules of switch statement.

B) Attempt any one: 05

c) Write a program to count the number of boys whose weight is less than 55kg and height is greater than 170cm.

d) Write a program to select and print the largest of the three numbers using nested if else……. statements.

Q2 A) Attempt any one: 05

a) Explain how jumping out of the program is done in C language.

b) Explain entry controlled loop and exit controlled loop in detail.

B) Attempt any one: 05

c) Write a program to evaluate the equation $y = x^n$, where n is non-negative integer.

d) Write a program using for loop to print the "power of 2" table for the power 0 to 15, both positive and negative.

Q3 A) Attempt any one: 05

a) Write a short note on data structures.

b) Discuss in detail two dimensional arrays with example.

B) Attempt any one: 05

c) Write a program to determine median for the given data.

d) Write a program for initialising large arrays when runtime is at 1.0

Q4 Fill in the blanks. 10

i. A multipath decision is a chain of if's in which the statement associated with each ________ is an if.

ii. A counter-controlled loop is called _______

iii. The empty ______ in the enter loop initiates a new line to print the next row.

iv. The unconditional ______ at the end, puts the computer in a permanent loop called _______

v. A complete set of values is called ______